

Holomorphic Discs in the Space of Oriented Lines via Mean Curvature Flow and Applications

Wilhelm Klingenberg, Durham University
(joint work with Brendan Guilfoyle, IT Tralee)

We introduce a metric \mathbb{G} on the space \mathbb{L} of oriented geodesics in Euclidean 3-space. It is of index $(2, 2)$. Together with the natural complex structure \mathbb{J} due to Nigel Hitchin and the classical symplectic structure Ω on this space, this endows \mathbb{L} with the structure of a neutral Kähler surface. The geometry of this space captures the geometry of C^1 -smooth surfaces in Euclidean 3-space via a correspondance that associates with S the family $\Sigma \subset \mathbb{L}$ of oriented Euclidean-normal lines to S .

Our results are as follows.

1. We establish long-time existence for those solutions of mean curvature flow for spacelike surfaces in (\mathbb{L}, \mathbb{G}) that remain in a fixed compact subset of \mathbb{L} .
2. Given a Lagrangian surface $\Sigma \subset \mathbb{L}$, we consider mean curvature flow for spacelike surfaces Σ_t in \mathbb{L} subject to three boundary conditions:
 - a) $\partial\Sigma_t \subset \Sigma$ for all t ,
 - b) the angle between Σ_t and Σ remains constant in time,
 - c) $T\partial\Sigma_t$ is holomorphic as $t \rightarrow \infty$.

In this situation, we prove that there exist times $t_j \rightarrow \infty$ such that Σ_{t_j} converges to a holomorphic curve Σ_{t_∞} .

3. We prove that the existence of Σ_{t_∞} as above implies a bound on the relative first chern class of the pair (\mathbb{L}, Σ) along the boundary of Σ_{t_∞} . This in turn implies a local index bound on the index of an isolated umbilic point of S . Here S arises as an integral surface of the family of lines that correspond to Σ in Euclidean 3-space.