MAX – PLANCK – GESELLSCHAFT

Informatik/Mathematik/Komplexe Systeme

Max-Planck-Institut für Mathematik Activity

Dynamical Numbers

Interplay between Dynamical Systems and Number Theory

Max-Planck-Institut für Mathematik
Bonn, Germany, 1 May - 31 July 2009

Organized by:

Bryna Kra, Sergiy F. Kolyada, Yuri I. Manin,
Martin Möller, Pieter Moree, Don Zagier, Tom Ward

Max-Planck-Institut für Mathematik, Bonn

&

Northwestern University - Institute of Mathematics, Kiev
University of East Anglia, Norwich

The scientific-presentation side of the activity consisted of two parts: the MPIM Seminar “Dynamical Numbers” was held in the period from 4 May to 19 July; from 27 to 29 July 2009 and the International conference “Dynamical Numbers Interplay between Dynamical Systems and Number Theory Dynamical Systems” was held in period from 20 to 24 July 2009.

The activity was proposed by Sergiy Kolyada. It was financed by the Max Planck Institute for Mathematics, the Hausdorff Center for Mathematics and the Clay Mathematics Institute. A good part of the organization was done by Sergiy Kolyada and Pieter Moree.

A direct outcome of the activity is a proceedings volume, which will be edited by Sergiy F. Kolyada, Yuri I. Manin, Martin Möller, Pieter Moree, Don Zagier and Tom Ward . The submission deadline is December 2009. The proceedings volume is expected to appear in the AMS Contemporary Mathematical series.

About 100 mathematicians were invited for the activity (including the International conference), some of whom, at own expense. They gave 65 lectures and also will take part in the proceedings of the activity. The speakers presented the following 17 countries: Austria, Canada(2), Czech Republic, Chile, China, Germany(3), France(7), Italy, Israel(6), Norway, Russia(2), Slovak Republic, Sweden, Switzerland, Ukraine, United Kingdom(7), United States (22).

MPIM ACTIVITY:

Dynamical Numbers: Interplay between Dynamical Systems and Number Theory

Bonn, Germany, 1 May – 31 July 2009

Friday 15 May 2009

14.00 - 15.00 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Pierre Moussa (CEA CENS Gif-sur-Yvette/MPI): **Localisation of algebraic integers and polynomial iterations**

Tuesday 19 May 2009

14.00 - 15.00 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Stefano Marmi (Scuola Normale Superiore di Pisa/MPI): **Real and complex Brjuno functions and their relation to dynamics**

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Thomas Ward (UEA Norwich/MPI): **Orbit growth and arithmetic, zeta functions and Dirichlet series**

Tuesday 2 June 2009

12.30 - 13.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Barak Weiss (Ben Gurion University/MPI): **Cohomology classes represented by measured foliations and applications**

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Alexander Blokh (U of Alabama/MPI): **On wandering sets for complex polynomials**

Thursday 4 June 2009

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Xiangdong Ye (U of Science and Technology China/MPI): **Towards a theory on independence in dynamics**

Friday 12 June 2009

14.00 - 15.00 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Michal Misiurewicz (IUPUI/MPI): **Omega-limit sets for spiral maps**

MPIM ACTIVITY:

Dynamical Numbers: Interplay between Dynamical Systems and Number Theory

Bonn, Germany, 1 May – 31 July 2009

Tuesday 16 June 2009

14.00 - 15.00 **Seminar on Algebra, Geometry and Physics** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Curtis T. McMullen (Harvard University/MPI): **Arithmetic chaos in real quadratic fields**

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Sergei Tabachnikov (Pennsylvania State University/MPI): **Outer billiards: results and open problems**

Friday 19 June 2009

12.30 - 13.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Thomas Ward (UEA Norwich/MPI): **Orbit counting for group actions**

14.00 - 15.0 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Ilya Kapovich (University of Illinois at Urbana-Champaign/MPI): **In search of geometry of Out(F_n): interplay between Outer space and the space of geodesic currents**

Tuesday 23 June 2009

12.30 - 13.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Zoran Sunik (Texas A & M University/MPI): **Forbidden patterns and groups acting on trees**

14.00 - 15.00 **Seminar on Algebra, Geometry and Physics** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Rostislav Grigorchuk (Texas A & M University/MPI): **Ergodicity of boundary actions, Hopf decomposition and Nielsen-Schreier method**

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Dmitry Kleinbock (Brandeis University/MPI): **"Almost all versus no" dichotomy in homogeneous dynamics and Diophantine approximation**

Friday 26 June 2009

14.00 - 15.00 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Oleg Ageev (Moscow State Technical University/MPI): **Counting multiplicities: Typical Rokhlin?**

15.30 - 16.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Alexandre Danilenko (ILTPE, Kharkov/MPI): **On the entropy concepts for infinite measure preserving transformations**

Tuesday 30 June 2009

14.00 - 15.00 **Seminar on Algebra, Geometry and Physics** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Yakov Pesin (Pennsylvania State Univ./MPI): **Thermodynamics of towers, liftability, and distribution of Borel's normal numbers**

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Chris Judge (Indiana University/MPI): **A 2-D cell complex associated to the orbits of the Teichmüller SL(2, ℝ) action**

Tuesday 7 July 2009

14.00 - 15.00 **Seminar on Algebra, Geometry and Physics** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Alexey Bondal (Steklov Mathematical Institute Moscow/MPI): **Orthogonal decompositions of sl(n,ℂ)**

Friday 10 July 2009

14.00 - 15.00 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Thierry Giordano (University of Ottawa/MPI): **Topological orbit equivalence of free, minimal actions of ℤ^d on the Cantor set**

15.30 - 16.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Pascal Hubert (Université Aix-Marseille III/MPI): **Translation surfaces of infinite area**

MPIM ACTIVITY:

Dynamical Numbers: Interplay between Dynamical Systems and Number Theory

Bonn, Germany, 1 May – 31 July 2009

Monday 13 July 2009

10.30 - 11.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Yves Benoist (U Paris-Sud/MPI): **Density and equidistribution of orbits in finite volume homogeneous spaces**

11.40 - 12.40 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Erwan Lanneau (Centre de Physique Théorique Marseille/MPI): **On the minimum dilatation of pseudo-Anosov homeomorphisms on surfaces of small genus**

15.00 - 16.00 **Topics in Topology** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Yves Benoist (U Paris-Sud/MPI): **Stationary probabilities on homogeneous spaces**

Tuesday 14 July 2009

11.40 - 12.40 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Svetlana Katok (Pennsylvania State U/MPI): **Structure of attractors for (a,b)-continued fractions, I**

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Tobias Jäger (Collège de France/MPI): **Linearisation of conservative toral homeomorphisms**

Wednesday 15 July 2009

10.30 - 11.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Alejandro Maass (U de Chile/MPI): **Nilsequences and a structure theorem for topological dynamical systems** (Joint work with B. Host and B. Kra)

11.40 - 12.40 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Petr Kurka (Charles U Prague/MPI): **Geometry of Möbius number systems**

14.15 - 15.15 **Number Theory Lunch Seminar** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Dieter Mayer (TU Clausthal/MPI): **Symbolic dynamics for the geodesic flow on Hecke surfaces and its transfer operator**

15.00 - 16.00 **Topics in Topology** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Svetlana Katok (Pennsylvania State U/MPI): **Symbolic dynamics of the geodesic flow on surfaces of constant negative curvature**

Thursday 16 July 2009

10.30 - 11.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Tien-Cuong Dinh (U Paris 06/MPI): **Equidistribution problems in complex dynamics**

11.40 - 12.40 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Manfred Einsiedler (Ohio State U, ETH Zürich and MPI): **Badly approximable inhomogeneous linear forms**

10.30 - 11.30 **Oberseminar Differentialgeometrie** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Jean-Christophe Yoccoz (Collège de France/MPI): **Uniformly hyperbolic SL(2,R) cocycles**

Friday 17 July 2009

10.30 - 11.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Ian Putnam (U Victoria, Canada/MPI): **A homology theory for basic sets**

11.40 - 12.40 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Fedor Pakovich (Ben Gurion U, Israel/MPI): **Solution of the polynomial moment problem**

14.00 - 15.00 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Radhakrishnan Nair (U Liverpool/MPI): **Polynomials and pointwise ergodic theorems**

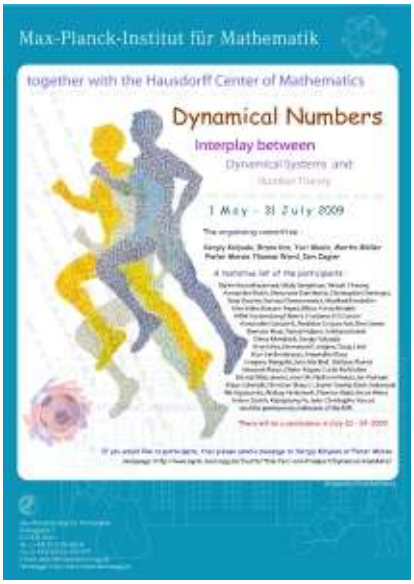
15.30 - 16.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Nikita Sidorov (U Manchester/MPI): **Beta-expansions and Bernoulli convolutions**

Clay Mathematics Institute
Hausdorff Center for Mathematics
Max Planck Institute for Mathematics

INTERNATIONAL CONFERENCE

Dynamical Numbers

Interplay between Dynamical Systems and Number Theory



Max-Planck-Institute for Mathematics
Bonn, Germany, 20-24 July 2009

Organizers:

David A. Ellwood, Sergiy F. Kolyada, Yuri I. Manin,
Martin Möller, Pieter Moree, Don Zagier

International Conference:

Dynamical Numbers: Interplay between Dynamical Systems and Number Theory

Bonn, Germany, 20-24 July 2009

FINAL PROGRAMME

Monday 20 July 2009

09.30 - 10.00 **Conference opening**
Welcome by organizers

Tea break

10.30 - 11.30 **Marmi, Stefano** (Scuola Normale Superiore di Pisa, Italy & MPI, Germany)
Roth type interval exchange maps

11.40 - 12.40 **Vershik, Anatoly** (St.Petersburg Department of Steklov Institute of Mathematics, Russia)
Arithmetic and dynamics

Lunch

14.00 - 14.50 **Bainbridge, Matthew** (University of Chicago, US & MPI, Germany)
Algebraically primitive Teichműller curves in genus three

15.00 – 16.00 **Deninger, Christopher** (Universität Münster & MPI, Germany)
Noncommutative Mahler measures and Ljapunov exponents

Tea break

16.30 – 17.30 **Marklof, Jens** (University of Bristol, UK & MPI, Germany)
The asymptotic distribution of Frobenius numbers and flows on homogeneous spaces

17.40 - 18.30 **Kontorovich, Alex** (Brown University, US)
On representations of integers in thin subgroups of SL(2,Z)

Tuesday 21 July 2009

09.00 - 09.50 **Bufetov, Alexander** (Rice University, US & MPI, Germany)
Limit theorems for translation flows

Tea break

10.30 - 11.30 **Glasner, Eli** (Tel-Aviv University, Israel & MPI, Germany)
A topological lens for a measure-preserving system

11.40 - 12.40 **Schmidt, Klaus** (University of Vienna, Austria & MPI, Germany)
Mahler measure, a dynamical number

Lunch

14.00 - 14.50 **Zieve, Michael** (Institute for Advanced Study, US)
Intersections of polynomial orbits, and a dynamical Mordell-Lang conjecture

15.00 - 16.00 **Einsiedler, Manfred** (Ohio State University, ETH Zürich & MPI, Germany)
Diophantine approximation on invariant fractals

Tea break

16.30 - 17.30 **Nevo, Amos** (Technion - Israel Institute of Technology & MPI, Germany)
Ergodic theory and lattice points

18.00 – **Open problems session**

Wednesday 22 July 2009

09.00 - 09.50 **Shapira, Uri** (Hebrew University of Jerusalem, Israel & MPI, Germany)
Applications of rigidity of higher rank hyperbolic actions

Tea break

10.30 - 11.30 **Lindenstrauss, Elon** (Hebrew University of Jerusalem, Israel & MPI, Germany)
Arithmetic combinatorics and effective equidistribution

11.40 - 12.40 **Ugarcovici, Ilie** (DePaul University, US & MPI, Germany)
Structure of attractors for (a,b)-continued fractions, II

Lunch

14.00 – 15.00 **Pollicott, Mark** (University of Warwick, UK & MPI, Germany)
Random matrix products and absolute continuity of the associated measures

15.10 – 16.00 **Solomyak, Boris** (University of Washington, US & MPI, Germany)
Dynamics of self-affine tilings and algebraic number theory

Tea break

20.00 – **Conference dinner**

Thursday 23 July 2009

09.00 - 09.50 **Wang, Zhiren** (Princeton University, US & HCM, Germany)
A quantitative result on actions of commutative toral automorphisms

Tea break

10.30 - 11.30 **Yoccoz, Jean-Christophe** (Collège de France & MPI, Germany)
Affine interval exchange maps

11.40 - 12.40 **Margulis, Gregori** (Yale University, US)
Distribution of values of indefinite quadratic forms at integral points

Lunch

14.00 – 14.50 **Jones, Rafe** (College of the Holy Cross, US)
Galois actions on preimage trees

15.00 – 16.00 **Ulcigrai, Corinna** (University of Bristol, UK & MPI, Germany)
Absence of mixing in locally Hamiltonian flows on surfaces

Tea break

16.30 – 17.30 **Oh, Hee** (Brown University, US & MPI, Germany)
Orbital counting for Thin groups

Friday 24 July 2009

09.00 - 09.50 **Snoha, Lubomir** (Matej Bel University, Slovakia & MPI, Germany)
Topology of minimal sets

Tea break

10.30 - 11.30 **Gorodnik, Alexander** (University of Bristol, UK & MPI, Germany)
Rigidity of actions on algebraic spaces

11.40 - 12.40 **Skau, Christian** (Norwegian University of Science and Technology & MPI, Germany)
Orbit equivalence and (ordered) K-theory

Lunch

14.00 - **Boat trip on the Rhine to Rolandseck**

Saturday 25 July 2009

Morning *Departure*

Tuesday 28 July 2009

16.30 - 17.30 **Dynamical Numbers Seminar**
Hörsaal MPI, Vivatsgasse 7
Roland Gunesch (Univ. Hamburg): **Orbit counting and the measure of maximal entropy on rank one manifolds**

Wednesday 29 July 2009

14.15 - 15.15 **Number Theory Lunch Seminar** jointly with Dynamical Numbers Seminar
Hörsaal MPI, Vivatsgasse 7
Jörg Schmeling (Lunds Universitet/MPI): **Diophantine approximation through orbits of some number-theoretical dynamical systems**

MPIM ACTIVITY:

Dynamical Numbers: Interplay between Dynamical Systems and Number Theory

Bonn, Germany, 1 May – 31 July 2009

PARTICIPANTS

1. Bainbridge, Matthew (University of Chicago, US)
2. Benoist, Yves (Université Paris-Sud, France)
3. Bernstein, Jozeph (Tel Aviv University, Israel)
4. Björglund, Michael (KTH, Sweden)
5. Blokh, Alexander (University of Alabama, US)
6. Bondal, Alexey (Steklov Mathematical Institute, Moscow, Russia)
7. Bruggeman, Roelof (University of Utrecht, Netherlands)
8. Bufetov, Alexander (Rice University, US)
9. Carminati, Carlo (Universita di Pisa, Italy)
10. Cellarosi, Francesco (Princeton University, US)
11. Chaika, Jon (Rice University, US)
12. Danilenko, Alexandre (ILT, Kharkov, Ukraine)
13. Deninger, Christopher (University of Münster, Germany)
14. Dinh, Tien-Cuong (Institut de Mathématiques de Jussieu,France)
15. Dirbak, Matus (Matej Bel University, Slovak Republic)
16. Einsiedler, Manfred (Ohio State University, US & ETH Zürich, Switzerland)
17. Eisner, Tanja (University of Tübingen, Germany)
18. Felshtyn, Alexander L. (University of Szczecin, Poland)
19. Frantzikinakis, Nikos (University of Memphis, US)
20. Gierzkiewicz, Anna (Jagiellonian University in Krakow, Poland)
21. Giordano, Thierry (University of Ottawa, Canada)
22. Glasner, Eli (Tel Aviv University, Israel)
23. Goetz, Arek (San Francisco State University, US)
24. Gorodnik, Alexander (University of Bristol, UK)
25. Grigorchuk, Rostislav (Texas A&M University, US)
26. Grigoriev, Dmitry (Université de Lille 1, France)
27. Gunesch, Roland (University of Hamburg)
28. Hosseini, Maryam (University of Guilan, Iran)
29. Hric, Roman (Matej Bel University, Slovak Republic)
30. Hubert, Pascal (Université Aix-Marseille III, France)
31. Jäger, Tobias (Collège de France)
32. Jones, Rafe (College of the Holy Cross, US)
33. Judge, Chris (Indiana University, US)
34. Kadyrov, Shirali (Ohio State University, US)
35. Kapovich, Ilya (University of Illinois at Urbana-Champaign, US)
36. Katok, Svetlana (Pennsylvania State University, US)
37. Kleinbock, Dima (Brandeis University, US)
38. Kolyada, Sergiy (Institute of Mathematics, Kiev, Ukraine)
39. Kontorovich, Alex (Brown University, US)
40. Kopei, Fabian (University of Münster, Germany)
41. Kurka, Petr (Charles University in Prague, Czech Republic)
42. Lanneau, Erwan (Centre de Physique Théorique Marseille, France)
43. Lindenstrauss, Elon (Hebrew University of Jerusalem, Israel)
44. Maass, Alejandro (Universidad de Chile, Santiago, Chile)
45. Marchese, Luca (Scuola Normale Superiore di Pisa, Italy)
46. Margulis, Gregory (Yale University, US)
47. Marklof, Jens (Bristol University, UK)
48. Marmi, Stefano (Scuola Normale Superiore di Pisa, Italy)
49. Matheus, Carlos (Collège de France)
50. Mayer, Dieter (TU Clausthal, Germany)

51.McMullen, Curtis (Harvard University, US)
52.Misiurewicz, Michal (IUPUI, US)
53.Möller, Martin (MPIM, Bonn, Germany)
54.Moree, Pieter (MPIM, Bonn, Germany)
55.Moroz, Boris (University Bonn, Germany)
56.Mozes, Shahar (Hebrew University of Jerusalem, Israel)
57.Moussa, Pierre (CEA-CENS, France)
58.Nair, Radhakrishman (University of Liverpool, UK)
59.Neunhäuserer, Jörg (Tech. Ed. Institute of West Macedonia, Greece)
60.Nevo, Amos (Israel Institute of Technology, Israel)
61.Oh, Hee (Brown University, US)
62.Pakovich, Fedor (Ben Gurion University of the Negev, Israel)
63.Pesin, Yakov (Pennsylvania State University, US)
64.Pollicott, Mark (Warwick University, UK)
65.Putnam, Ian F. (University of Victoria, Canada)
66.Raulf, Nikole (Université de Lille 1, France)
67.Rybak, Oleksandr (Institute of Mathematics, Kiev, Ukraine)
68.Sauzin, David (CNRS - Institut de Mécanique Céleste, Paris, France)
69.Scharlau, Rudolf (Dortmund University, Germany)
70.Schmeling, Jörg (Lund Institute of Technology, Sweden)
71.Schmidt, Klaus (ESI, Austria)
72.Schmoll, Martin (Clemson University, US)
73.Shapira, Uri (Hebrew University of Jerusalem, Israel)
74.Sidorov, Nikita (University of Manchester, UK)
75.Skau, Christian F. (NTNU, Dragvoll, Norway)
76.Snoha, L'ubomir (Matej Bel University, Slovak Republic)
77.Solomyak, Boris (University of Washington, US)
78.Spitalsky, Vladimir (Matej Bel University, Slovak Republic)
79.Sun, Hae-Sang (Korea Institute for Advanced Study, Korea)
80.Sunik, Zoran (Texas A&M University, US)
81.Tabachnikov, Sergei (Pennsylvania State University, US)
82.Tiozzo, Giulio (Harvard University, US)
83.Troitskiy, Evgeni (Moscow State University, Russia)
84.Turchin, Victor (Kansas State University, US)
85.Ugarcovici, Ilie (DePaul University, US)
86.Ulcigrai, Corinna (University of Bristol, UK)
87.Ustian, Alex (Ohio State University, US)
88.Verbitskiy, Evgeny (Philips Research, Eindhoven, The Netherlands)
89.Vershik, Anatoly (PDMI, Russia)
90.Wang, Zhiren (Princeton University, US)
91.Ward, Thomas (University of East Anglia, UK)
92.Weiss, Barak (Ben Gurion University, Israel)
93.Weiss, Christian (MPIM, Bonn, Germany)
94.Winn, Brian (Loughborough University, UK)
95.Ye, Xiangdong (Hefei University, China)
96.Yoccoz, Jean-Christophe (Collège de France)
97.Zagier, Don (MPIM, Bonn, Germany & Collège de France)
98.Zieve, Michael (Institute for Advanced Study, US)
99.Zorich, Anton (Université de Rennes, France)

MAX – PLANCK – GESELLSCHAFT

Informatik/Mathematik/Komplexe Systeme

Max-Planck-Institut für Mathematik Activity

Dynamical Numbers

Interplay between Dynamical Systems and Number Theory

ABSTRACTS

Max-Planck-Institut für Mathematik
Bonn, Germany, 1 May - 31 July 2009

Organized by:

Bryna Kra, Sergiy F. Kolyada, Yuri I. Manin,
Martin Möller, Pieter Moree, Don Zagier, Tom Ward

Max-Planck-Institut für Mathematik, Bonn

&

Northwestern University - Institute of Mathematics, Kiev
University of East Anglia, Norwich

Pierre Moussa (CEA CENS Gif-sur-Yvette/MPI)

Localisation of algebraic integers and polynomial iterations

We consider the problem of finding all algebraic integers which belong to a domain of the complex plane together with their conjugates. Classic results go back to Kronecker and Lehmer. We show how these results can be extended to domains attached to iterations of polynomials with integer coefficients (Julia sets), as well as some generalisations.

Stefano Marmi (Scuola Normale Superiore di Pisa/MPI)

Brjuno functions and their relation to dynamics

When an irrational rotation is analytically perturbed, it is a natural question to ask whether or not there exists a neighborhood of the fixed point where the dynamics looks like the unperturbed case.More precisely, does there exist a local holomorphic coordinate for which the perturbed transformation is expressed as an ordinary rotation? When the rotation number satisfies the Brjuno condition such a coordinate exists in a domain called a Siegel disk. The Brjuno function tells more: it gives an estimate of minus the logarithm of the size of the Siegel disks as a function of the rotation number. A similar result also holds for certain area-preserving maps. We will describe the functional equation satisfied by the Brjuno function as well as some properties of its complexification. The talk is based on joint work with Pierre Moussa and Jean-Christophe Yoccoz.

Thomas Ward (UEA Norwich/MPI)

Orbit growth and arithmetic, zeta functions and Dirichlet series

We describe results on the asymptotic growth in closed orbits for compact group automorphisms. This involves a parametrized poset of problems, at one end of which there is a zeta function with meromorphic extension beyond its radius of convergence, at the other a Dirichlet series with reasonable behaviour on the critical line. Further into the poset from one end, the zeta function has a natural boundary, and from the other the Dirichlet series has complex behaviour on the critical line. Both suggest a different notion of adelic meromorphic functions which would permit Tauberian proofs of the results arrived at by cumbersome elementary means. (Joint work with Everest, Miles, Stevens).

Barak Weiss (Ben Gurion University/MPI)

Cohomology classes represented by measured foliations and applications

A translation surface of type (S, Sigma) gives rise to its horizontal and measured foliations on S with singularities in Sigma, and these in turn give rise to cohomology classes in H1(S, \Sigma). In joint work with Yair Minsky, for a fixed measured foliation F we characterize the set of cohomology classes b for which there is a translation surface structure with horizontal and vertical foliations F and G, such that b=[G]. This extends previous work of Thurston and Sullivan.

We present two applications. The first is an analogue of Mahler's question for interval exchanges - we prove that for the permutation sigma(i)=d+1-i, for almost every s, the interval exchange determined by sigma and (s, s2, ..., s^d) is uniquely ergodic. The second concerns the operation "real REL" in which singularities are moved horizontally with respect to each other, fixing absolute periods. We show that on a set of full measure, the real REL foliation is the orbit foliation for the action of the group B semidirect W, where B is the group of triangular matrices in SL(2, R) and W is a vector space of dimension |Sigma|-1.

Alexander Blokh (U of Alabama/MPI)

On wandering sets for complex polynomials

We first give an overview of a phenomenon of wandering sets in dynamical systems. Then we concentrate upon wandering branch-continua in polynomial Julia sets (continua which cut the Julia set in more than 2 pieces). As their existence is related to that of specific critical points of the polynomial, and because the same can be said about non-repelling cycles, we succeed in obtaining a combined estimate from above on the number of such wandering continua together with the number of non-repelling cycles of the map. This extends the Fatou-Shishikura inequality in the polynomial case.

Xiangdong Ye (U of Science and Technology China/MPI)

Towards a theory on independence in dynamics

Entropy, sequence entropy and un-tame tuples are well studied subjects in topological dynamics and ergodic theory. Recent results by Huang-Ye and Kerr-Li tell us that these notions can be characterized by using so called independence sets (or interpolating sets) which are collections of some subsets of natural numbers (we call them families).

In this talk for a given family we associate a dynamical system which has independence sets in the family. We try to understand the dynamical properties for systems defined in this way for some important families. It is a joint work with W. Huang and H.F. Li.

Michal Misiurewicz (IUPUI/MPI)

Omega-limit sets for spiral maps

In the late 1950's, using computers in the Los Alamos National Laboratory, Stanislaw Ulam and Paul Stein performed a comprehensive research on a class of quadratic maps of the 2-dimensional simplex Δ to itself. Those maps arise in the theory of population genetics. One of them has the behavior much different than the 96 other ones. We call it the Stein-Ulam Spiral map. In 1972, S.Vallander asked whether the ω-limit set of any interior point of Δ, except its center, is equal to the boundary of Δ. We prove that this is the case for the points from a residual subset of Δ. On the other hand, we show that for any closed invariant subset E of the boundary of Δ intersecting all three sides of Δ, the set of points having E as the ω-limit set is relatively large.

The proof of the above results requires 14 pages of tricky estimates. One can ask whether those estimates are necessary. Maybe similar results hold under purely topological assumptions? It turns out that the answer is negative - one can construct similar examples of homeomorphisms with completely different behavior. There are examples where the family of all ω-limit sets for a given map is a prescribed finite or countable family (consisting of sets that can be ω-limit). In particular, there exists a homeomorphism of a closed disk for which the set of all ω-limit sets of points is not closed in the space of all invariant closed nonempty subsets of the disk with the Hausdorff metric. This is in a sharp contrast with the results about continuous interval maps and continuous circle maps. The first part is a joint work with Krzysztof Baranski, the second part is a joint work with Bruce Kitchens.

Sergei Tabachnikov (Pennsylvania State University/MPI)

Outer billiards: results and open problems

I shall survey three problems on a lesser known class of billiards, the outer (a.k.a. Dual) billiards. The first is the existence of periodic trajectories for polygonal outer billiards, a theorem in affine, and an open problem in the hyperbolic plane (the problem is open and very hard for polygonal inner billiards!). The second is a version of the Birkhoff conjecture for outer billiards, which is a theorem in the algebraic setting, and a conjecture in the smooth one. The third is Moser's problem whether the orbits of polygonal outer billiard may escape to infinity; this problem was recently solved (in the affirmative) by R. Schwartz.

Thomas Ward (UEA Norwich/MPI)

Orbit counting for group actions

We find asymptotics for orbit growth in a family of group actions generated by commuting automorphisms. A dichotomy is found in rank exceeding one between systems with or without a preferred direction with an excess of periodic orbits. [This is joint work with Richard Miles].

Ilya Kapovich (University of Illinois at Urbana-Champaign/MPI)

In search of geometry of Out(F_n): interplay between Outer space and the space of geodesic currents

We explore the notion of a Bonahon-type geometric intersection number between points of the Culler-Vogtmann Outer space and geodesic currents on free groups. Applications include studying free group analogues of the curve complex, "filling" elements in free groups and Schottki-type subgroups of Out(F_N).

Zoran Sunik (Texas A & M University/MPI)

Forbidden patterns and groups acting on trees

Basic notions from one-dimensional one-sided symbolic dynamics fit nicely in the extended setting of group actions on rooted regular trees. In this talk we discuss connections relating closed self-similar groups of tree automorphisms, forbidden patterns, automaton groups, and branch groups.

Rostislav Grigorchuk (Texas A & M University/MPI)

Ergodicity of boundary actions, Hopf decomposition and Nielsen-Schreier method

I will speak about ergodic properties (ergodicity and conservativity) of the action of a subgroup of a free group on its boundary with respect to the uniform measure. The Hopf decomposition of the boundary action will be described in terms of Nielsen-Schreier theory (one of the main tools in combinatorial group theory). Growth and cogrowth will be used to characterize conservativity and dissipativity, and amenability and the Liouville property will be mentioned to illustrate some extreme cases. This is joint work with V.Kaimanovich and T.Nagnibeda.

Oleg Ageev (Moscow State Technical University/MPI)

Counting multiplicities: Typical Rokhlin?

Restricting ourselves to typical (=residual) dynamical systems/unitary representations, we treat their spectral invariants. Traditionally, the "simplest" part in the spectrum is discrete. We find the exact description of the discrete part for a typical dynamical system/unitary representation of every countable group. In particular, we get its "rigidity", i.e. its independence from our choice of some representative of a typical set. It is suggested to study possible groups where the notion "typical" could be trivial in the sense that the typical element is unique up to an isomorphism. We prove that for (T)-groups it is equivalent to the density of finitely dimensional representations in the unitary dual. We will also discuss both related natural questions and possible applications.

Dmitry Kleinbock (Brandeis University/MPI)

"Almost all versus no" dichotomy in homogeneous dynamics and Diophantine approximation

Several years ago I proved (in an involved and somewhat mysterious way) the following theorem: suppose M is an analytic submanifold of R^n which contains a not very well approximable vector; then almost all its vectors are not very well approximable. Recently I found a simple argument establishing this and other more general results involving behavior of typical trajectories of dynamical systems. The proof uses measure estimates on the space of lattices.

Alexandre Danilenko (ILTPE, Kharkov/MPI)

On the entropy concepts for infinite measure preserving transformations

There are several concepts of entropy for dynamical systems with infinite invariant measure. I will talk about Krengel entropy, Parry entropy and Poisson entropy. I am going to shed light on progress achieved in recent works of Janvresse, Mayerovitch, Roy and de la Rue, Danilenko and Rudolph, Aaronson and Park about properties of these entropies and interrelations among them. Some open problems will be stated.

Yakov Pesin (Pennsylvania State Univ./MPI)

Thermodynamics of towers, liftability, and distribution of Borel's normal numbers

Thermodynamical formalism of statistical physics is a collection of results aimed at producing some "natural" invariant measures with strong ergodic properties including Sinai-Ruelle-Bowen measures (absolutely continuous invarinant measures in the one-dimensional case), measures of maximal dimension and entropy. In particular, one can study the pressure function, which plays a crucial role in multiftactal analysis and in applications to number theory, including fine distribution of Borel's normal numbers. In the classical situations (Anosov maps, hyperbolic attractors, etc.) thermodynamical formalism can be effected using symbolic representations of these systems via subshifts of finite type. Many non-classical situations (e.g., one-dimensional unimodal maps and Henon attractors) can be handled using symbolic representations via towers whose base is the Bernoulli shift on a countable set of states. I will describe general tower constructions and present some recent results on thermodynamics of the corresponding systems and in particular, on the study of the pressure function. The principal new phenomenon is that one may have to reduce the class of invariant measures under consideration to the so-called liftable measures. I will describe these measures and discuss the associated liftability problem.

Chris Judge (Indiana University/MPI)

A 2-D cell complex associated to the orbits of the Teichmüller $SL(2, \mathbb{R})$ action

The orbits of a natural SL(2, ℝ) action on the cotangent space to Riemann moduli space project to totally geodesic (real) surfaces in the moduli space with respect to the Teichmüller metric. The orbit projections that have finite area are dense in moduli space and so one is led to think of moduli space as being like an arithmetic lattice. In general, very little is known about these orbits, for example, their topological types. In this talk, I will describe a polyhedral cell complex that encodes the geometry and dynamics of each orbit.

Alexey Bondal (Steklov Mathematical Institute Moscow/MPI)

Orthogonal decompositions of sl(n,C)

We will discuss the problem of orthogonal decompositions of a Lie algebra as a sum of Cartan subalgebras. We will show how the problem is related to representation theory of Hecke algebras on graphs, to harmonic analysis on graphs, to a problem of quantum information and to noncommutative geometry. A 4-dimensional family of orthogonal pairs of Cartan subalgebras will be described.

Thierry Giordano (University of Ottawa/MPI)

Topological orbit equivalence of free, minimal actions of Z^d on the Cantor set

In 1959, H. Dye introduced the notion of orbit equivalence and proved that any two ergodic finite measure-preserving transformations on a Lebesgue space are orbit equivalent. He also conjectured that an arbitrary action of a discrete amenable group is orbit equivalent to a Z-action. This conjecture was proved by Ornstein and Weiss and its most general case by Connes, Feldman and Weiss by establishing that an amenable non-singular countable equivalence relation R can be generated by a single transformation, or equivalently is hyperfinite, i.e., R is up to a null set, a countable increasing union of finite equivalence relations.

In the Borel case, Weiss proved that actions of Z^d are (orbit equivalent to) hyperfinite Borel equivalence relations, whose classification was obtained by Dougherty, Jackson and Kechris. In 1995, Giordano, Putnam and Skau proved that minimal Z-actions on the Cantor set were orbit equivalent to approximately finite (AF) relations and their classification was given. In this talk I will indicate the main steps of the proof of the general result obtained in a joint effort with H. Matui, I. Putnam and C. Skau and whose statement is the following:

Theorem. Any minimal, free Z^d-action on the Cantor set is affable (i.e., orbit equivalent to AF-relations).

Pascal Hubert (Université Aix-Marseille III/MPI)

Translation surfaces of infinite area

In this talk, I will briefly review some classical results by Veech about square tiled surfaces (that are some translation surfaces with remarkable properties). Then, I will explain some attempts to generalize some results in the case of infinite area. I will concentrate on the case of Z and Z^2 covers translation surfaces of finite area.

Yves Benoist (U Paris-Sud/MPI)

Density and equidistribution of orbits in finite volume homogeneous spaces

Let G be a Lie group, X be a G-homogeneous space of finite volume, and H be a closed subgroup of G. Are all the infinite H-orbits dense in X? Does every sequence of distinct finite H-orbits become equidistributed in X? I will focus on a joint work with J.F. Quint answering these questions when G is simple and H is Zariski dense with yes.

Svetlana Katok (Pennsylvania State U/MPI)

Structure of attractors for (a,b)-continued fractions, I

I will discuss one-dimensional maps related to a family of (a,b)-continued fractions, suggested for consideration by Don Zagier, and give a sufficient condition for validity of the Reduction theory conjecture that states that the associated natural extension maps have attractors with finite rectangular structure where every point of the plane is mapped after finitely many iterations. I will show that the structure of these attractors can be "computed" from the data (a,b), and give a dynamical interpretation of the "reduction theory" that underlines these constructions. The structure of the set of parameters (a,b) for which the conjecture is not valid will be discussed in a subsequent talk. This is a joint work with Ilie Ugarcovici.

Tobias Jäger (Collège de France/MPI):

Linearisation of conservative toral homeomorphisms

One of earliest, but still one of the most elegant, classification results in Dynamical Systems is Poincaré's classification of the dynamics of orientation-preserving circle homeomorphisms. Its core part is an equivalent characterization for the existence of a semi-conjugacy to an irrational rotation: the irrationality of the rotation number. We present an analogue of this result for area-preserving homeomorphisms of the two-torus that are homotopic to the identity. These are semi-conjugate to a two-dimensional irrational rotation if and only if they have a unique totally irrational rotation vector and the property of bounded mean motion. The latter means that the speed of convergence of the rotation vector is sufficiently fast (that is, bounded by $1/n \cdot$ constant). The proof is based on the concept of a circloid, which also leads to a number of related results.

Alejandro Maass (U de Chile/MPI)

Nilsequences and a structure theorem for topological dynamical systems

We characterize inverse limits of nilsystems in topological dynamics, via a structure theorem for topological dynamical systems that is an analog of the structure theorem for measure preserving systems. We provide two applications of the structure. The first is to nilsequences, which have played an important role in recent developments in ergodic theory and additive combinatorics; we give a characterization that detects if a given sequence is a nilsequence by only testing properties locally, meaning on finite intervals. The second application is the construction of the maximal nilfactor of any order in a distal minimal topological dynamical system. We show that this factor can be defined via a certain generalization of the regionally proximal relation that is used to produce the maximal equicontinuous factor and corresponds to the case of order 1. (Joint work with B. Host and B. Kra).

Petr Kurka (Charles U Prague/MPI)

Geometry of Möbius number systems

A Möbius iterative system is a system of real Möbius transformations indexed by a finite alphabet. A Möbius number system is given by a subshift such that each word of the subshift represents a real number, and this representation is continuous and surjective. We give several sufficient conditions on a subshift to form a Möbius number system. We show several examples based on continued fractions. We consider polygonal number systems whose transformations tessellate the hyperbolic space by regular polygons. We introduce the Biternary system which is based on a Fuchsian group whose fundamental domain is a rectangle with ideal vertices.

MPIM ACTIVITY:

Dynamical Numbers: Interplay between Dynamical Systems and Number Theory

Bonn, Germany, 1 May – 31 July 2009

Tien-Cuong Dinh (U Paris 06/MPI):
Equidistribution problems in complex dynamics

Let f be a non-invertible holomorphic endomorphism of the complex projective space CP^k. There is a maximal proper analytic subset E of CP^k, possibly empty, which is totally invariant under f. The preimages of a point outside E under the sequence of iterates of f are equidistributed with respect to a canonical invariant measure. We also discuss equidistribution problems for the preimages of analytic sets in CP^k. This is joint work with Nessim Sibony.

Manfred Einsiedler (Ohio State U, ETH Zürich and MPI)
Badly approximable inhomogeneous linear forms

We show that various subsets of badly approximable systems of affine forms are winning in the sense of Schmidt games and thus have full Hausdorff dimension and the countable intersection property. This is joint work with Jim Tseng.

Ian Putnam (U Victoria, Canada/MPI)
A homology theory for basic sets

We develop a kind of homology theory for the basic sets of Smale's Axiom A systems. The key ingredients are a variation of Bowen's theorem showing the existence of a factor map from a shift of finite type onto a basic set (via Markov partitions) and Krieger's dimension group invariant for shifts of finite type.

Fedor Pakovich (Ben Gurion U, Israel/MPI)
Solution of the polynomial moment problem

About ten years ago in the series of papers of M. Briskin, J. -P. Francoise and Y. Yomdin the following problem arose: "for a given complex polynomial P(z) and complex numbers a,b to describe polynomials q(z) orthogonal to all powers of P(z) on the segment [a,b]". Posed initially as an intermediate step of a broad research program concerning Poincare center-focus problem, the polynomial moment problem turned out to be a quite subtle question unexpectedly related to Galois theory and Representation Theory over $\mathbb{Q}$. In the talk we outline the recent solution of the polynomial moment problem obtained by F.Pakovich and M.Muzychuk and discuss different applications and generalizations.

Radhakrishnan Nair (U Liverpool/MPI)
Polynomials and pointwise ergodic theorems

The talk will give a survey of pointwise subsequence ergodic theorems and their number theoretic applications starting with J. Bourgain's famous theorem for squares.

Nikita Sidorov (U Manchester/MPI)
Beta-expansions and Bernoulli convolutions

It is well known that for any q in (1,2) each x in [0,1] can be expanded into a series in decreasing powers of q with coefficients 0,1 (a q-expansion). In fact, it is also known a.e. x has a continuum of q-expansions. In my talk I am going to discuss the rate of growth for such a continuum; in particular, if q is a Pisot number (an algebraic integer >1 whose conjugates are less than 1 in modulus), then this continuum has the same growth exponent for a.e. x. This topic is closely related to Bernoulli convolutions (which are a certain class of measures supported by the interval) – namely, to the dimension theory of such measures. In particular, I will explain how to obtain a reasonable lower bound for the Garsia entropy of any Bernoulli convolution parametrized by a Pisot number via q-expansions. My talk is based on my recent papers with De-Jun Feng and Kevin Hare.

MPIM ACTIVITY:
Dynamical Numbers: Interplay between Dynamical Systems and Number Theory
Bonn, Germany, 1 May – 31 July 2009

Roland Gunesch (Univ. Hamburg)

Orbit counting and the measure of maximal entropy on rank one manifolds

I will show how to prove a precise asypmtotic formula for the number of closed orbits of the geodesic flow on manifolds of non-positive curvature and rank one. This generalizes well-known results of G. A. Margulis to some spaces without uniform hyperbolicity. It also strengthens previous results of G. Knieper. I will also explain how the Margulis construction of the measure of maximal entropy can be carried out in non-positive curvature.

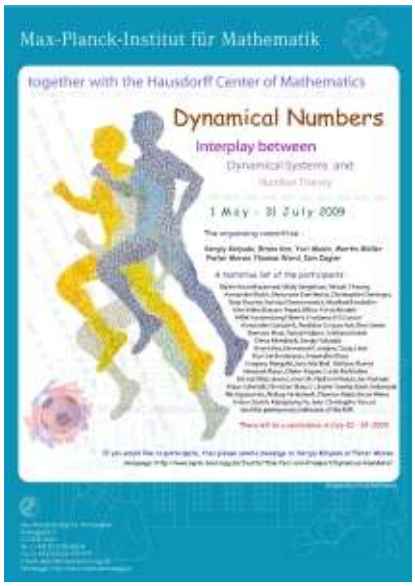
Clay Mathematics Institute
Hausdorff Center for Mathematics
Max Planck Institute for Mathematics

INTERNATIONAL CONFERENCE

Dynamical Numbers

Interplay between Dynamical Systems and Number Theory

ABSTRACTS



Max-Planck-Institute for Mathematics
Bonn, Germany, 20-24 July 2009

Organizers:

David A. Ellwood, Sergiy F. Kolyada, Yuri I. Manin,
Martin Möller, Pieter Moree, Don Zagier

Bainbridge, Matthew (University of Chicago, US & MPI, Germany)

Algebraically primitive Teichműller curves in genus three

A Teichműller curve is a finite-volume Riemann surface C = H/G together with an isometric immersion C --> M_g, where M_g is the moduli space of genus g Riemann surfaces, equipped with the Teichműller metric. The trace field F(C) is the number field obtained by adjoining to $\mathbb{Q}$ the traces of the elements of G (a subgroup of SL_2($\mathbb{R}$)). It is known that F(C) has degree at most g, and C is said to be algebraically primitive if F(C) has degree g.

McMullen and Calta have independently discovered an infinite family of algebraically primitive Teichműller curves in M_2. It is an open question whether there are infinitely many algebraically primitive Teichműller curves in M_g for any g>2. In this talk I will present some recent theorems and computer evidence indicating that there should be only finitely many algebraically primitive Teichműller curves in M_3. This is joint work with Martin Möller.

Bufetov, Alexander (Rice University, US & MPI, Germany)

Limit theorems for translation flows

Consider a compact oriented surface of genus at least two endowed with a holomorphic one-form. The real and the imaginary parts of the one-form define two foliations on the surface, and each foliation defines an area-preserving translation flow. By a Theorem of H.Masur and W.Veech, for a generic surface these flows are ergodic. The talk will be devoted to the speed of convergence in the ergodic theorem for translation flows. The main result, which extends earlier work of A.Zorich and G.Forni, is a multiplicative asymptotic expansion for time averages of Lipschitz functions. The argument, close in spirit to that of G.Forni, proceeds by approximation of ergodic integrals by special holonomy-invariant Hoelder cocycles on trajectories of the flows. In particular, a simple explicit construction is given for G. Forni's invariant distributions of translation flows. Generically, the dimension of the space of holonomy-invariant Hoelder cocycles is equal to the genus of the surface, and the ergodic integral of a Lipschitz function can be approximated by such a cocycle up to terms growing slower than any power of the time. The renormalization effectuated by the Teichmüller geodesic flow on the space of holonomy-invariant Hölder cocycles allows one also to obtain limit theorems for translation flows: it is proved that along certain sequences of times ergodic integrals, normalized to have variance one, converge in distribution to a non-degenerate compactly supported measure. The argument uses a symbolic representation of translation flows as suspension flows over Vershik's automorphisms, a construction similar to one proposed by S.Ito.

Deninger, Christopher (Universität Münster & MPI, Germany)

Noncommutative Mahler measures and Ljapunov exponents

For an ergodic measure preserving action on a probability space, consider the corresponding crossed product von Neumann algebra. We calculate the Fuglede-Kadison determinant for the "algebraic" operators in this von Neumann algebra in terms of the Ljapunov exponents of an associated measurable cocycle. The proof is based on the theory of the Brown measure, recent work of Dykema and Schultz and an analysis of the proof of the multiplicative ergodic theorem. As an application one obtains formulas for the Fuglede-Kadison determinant of noncommutative polynomials in the von Neumann algebra of the discrete Heisenberg group previously obtained by Lind and Schmidt via entropy considerations. These determinants can also be viewed as non-commutative generalizations of the Mahler measures prominent from number theory and hence as non-commutative heights.

Einsiedler, Manfred (Ohio State University, US & ETH Zürich, Switzerland & MPI, Germany)

Diophantine approximation on invariant fractals

We start by recalling the conjectures and the known theorems regarding the dynamics of diagonalizable subgroup actions on homogeneous spaces. In particular, we will discuss the question which probability measures are invariant under the full diagonal subgroup A on $SL(n,\mathbb{Z})\backslash SL(n,\mathbb{R})$ for $n>2$ and the partial classification of such measures due to Katok, Lindenstrauss, and myself. In the same joint work this theorem was applied to Littlewood's conjecture on multiplicative Diophantine approximation. Somewhat surprisingly, the same (or closely related) theorems also help to understand the Diophantine properties of typical points on dynamically defined fractals like the regular middle-third Cantor set. We will discuss why almost every point on the Cantor set is well approximable and related theorems. This is joint work with Fishman and Shapira.

Glasner, Eli (Tel-Aviv University, Israel & MPI, Germany)

A topological lens for a measure-preserving system

I will describe a functor which associates to every measure preserving system $(X,\mathcal{B},\mu,T)$ a topological system $(C^2(\mu),\check{T})$ defined on the space of 2-fold couplings of μ, called the *topological lens* of T. As we shall see the topological lens 'magnifies' the basic measure dynamical properties of T in terms of the corresponding topological properties of $\check{T}$. Some of our main results are as follows: (i) T is weakly mixing iff $\check{T}$ is topologically transitive (iff it is topologically weakly mixing); (ii) T has zero entropy iff $\check{T}$ has zero topological entropy, and T has positive entropy iff $\check{T}$ has infinite topological entropy; (iii) For T a K-system, the topological lens is a P-system (i.e. it is infinite, topologically transitive and the set of periodic points is dense; such sytems are also called chaotic in the sense of Devaney). This is joint work with M. Lemanczyk and B. Weiss.

Gorodnik, Alexander (University of Bristol, UK & MPI, Germany)

Rigidity of actions on algebraic spaces

In this talk we will be interested in showing that objects that apriori are defined only in the measurable category have to respect geometric/algebraic structure of the space. In particular, we discuss a classification of measurable morphisms and factors for dynamical systems on general algebraic varieties, which is analogous to the Margulis' classification of factors of flag varieties. This is joint work with Bader, Furman, and Weiss.

Jones, Rafe (College of the Holy Cross, US)

Galois actions on preimage trees

In a discrete dynamical system, the set of all points eventually mapping to a given point P forms a rooted tree in a natural way. When the system is given by iteration of a rational function (or more generally a morphism) of degree d, and both the map and P are defined over a number field K, then there is a natural action of the absolute Galois group of K on the rooted tree of preimages of P. This action is known as an arboreal Galois representation. This talk presents a survey of known results about such representations, in particular addressing the question of when the action has finite index in the full automorphism group of the preimage tree. I will also give some applications to arithmetic questions about certain dynamical systems, and discuss analogies with the theory of linear Galois representations attached to abelian varieties.

Kontorovich, Alex (Brown University, US)

On representations of integers in thin subgroups of SL(2,$\mathbb{Z}$)

The Affine Linear Sieve extends sieve methods to thin orbits of non-abelian group actions. The fundamental work of Bourgain-Gamburd-Sarnak showed in great generality that one may obtain an infinitude of R-almost primes (numbers with at most R prime factors), but the method does not specify R. Explicit values of R were obtained in some specific situations by Kontorovich and Kontorovich-Oh, but unconditionally are still in the teens (i.e. R=14). In recent joint work with Jean Bourgain, we obtain an infinitude of actual primes in such an orbit. We show that not only do infinitely many primes appear, but almost every number appears, as long as it is not excluded by congruence conditions; moreover the number of exceptions is a power savings. The main ingredient is the circle method.

Lindenstrauss, Elon (Hebrew University of Jerusalem, Israel & MPI, Germany)

Arithmetic combinatorics and effective equidistribution

A remarkable development in recent years has been a dramatic advance in our understanding of quantitative equidistribution thanks to the techniques of arithmetic combinatorics. I will present one result in this direction by Bourgain, Furman, Mozes and myself for random walks on the torus by endomorphisms, and explain how it relates to other results in the field, in particular an exponential sum estimate by Bourgain, Glibichuk and Konyagin.

Margulis, Gregori (Yale University, US)

Distribution of values of indefinite quadratic forms at integral points

I will describe different approaches to to the study of the aymptotic behavior of the number of integral solutions of the inequality a<Q(x)<b in "large" domains where Q is an indefinite irrational quadtaric form in n>2 variables.

Marklof, Jens (University of Bristol, UK & MPI, Germany)

The asymptotic distribution of Frobenius numbers and flows on homogeneous spaces

The Frobenius number F(a) of an integer vector a with positive coprime coefficients is defined as the largest number that does not have a representation as a positive integer linear combination of the coefficients of a. We show that if a is taken to be random in an expanding d-dimensional domain, then F(a) has a limit distribution, which is given by the probability distribution for the covering radius of a certain simplex with respect to a (d-1)-dimensional random lattice. This result extends recent studies for d=3 by Arnold, Bourgain-Sinai and Shur-Sinai-Ustinov. The key features of our approach are (a) a novel interpretation of the Frobenius number in terms of the dynamics of a certain group action on the space of d-dimensional lattices, and (b) an equidistribution theorem for a multidimensional Farey sequence on closed horospheres.

Nevo, Amos (Technion - Israel Institute of Technology & MPI, Germany)

Ergodic theory and lattice points

We will begin by describing a general lattice point counting result based on ergodic theoretic methods. We will then expose some of its consequences for counting integral, rational and (almost) prime points on certain homogeneous algebraic varieties.

Marmi, Stefano (Scuola Normale Superiore di Pisa, Italy & MPI, Germany)

Roth type interval exchange maps

In 1955 Roth proved that every irrational algebraic number is of Roth type, namely has irrationality measure equal to 2. Lesser known is that Roth type numbers have an equivalent characterization in terms of the regularity of the soultions of the cohomological equation associated to circle rotations. Interval exchange maps generalize rotations and are characterized by combinatorial and metric data. The analysis of first return times on an interval (renormalisation) leads to a remarkable extension of the classical continued fraction algorithm (Rauzy, Veech, Zorich). We exhibit an explicit full measure class of minimal interval exchange maps for which the cohomological equation has a bounded solution provided that the datum belongs to a finite codimension subspace of the space of functions with derivative of bounded variation on each interval. The class of interval exchange maps is characterized in terms of a diophantine condition of Roth type imposed to an acceleration of the Rauzy-Veech-Zorich continued fraction expansion associated to the map. This talk is based on joint work with Pierre Moussa and Jean-Christophe Yoccoz.

Oh, Hee (Brown University, US & MPI, Germany)

Orbital counting for Thin groups

For a given homogeneous variety V of a semisimple algebraic group G defined over $\mathbb{Q}$, and for a subgroup Γ of G($\mathbb{Z}$), the orbital counting problem for Γ refers to the question of understanding the asymptotic number of integral vectors in the orbit of v_0 under Γ (for some fixed v_0 in V($\mathbb{Z}$)) of size at most T as T tends to infinity. This question has been extensively studied when Γ is of finite index in G($\mathbb{Z}$). On the other hand, the orbital counting problem for a thin group (that is, when Γ is of infinite index in G($\mathbb{Z}$)) is very far from being known in general. I will discuss a recent work with Nimish Shah on this question when V is a quadric defined by an integral quadratic form of signature (n,1), with n at least 2. If time permits, I will also explain an application to a hyperbolic circle packing problem.

Pollicott, Mark (University of Warwick, UK & MPI, Germany)

Random matrix products and absolute continuity of the associated measures

In this talk we will consider random products of a finite set of matrices in SL(2, $\mathbb{R}$). For positive matrices we present a new algorithm for accurately evaluating their Lyapunov exponents and the Furstenberg (stationionary) measure. In joint work with B. Barany and K. Simon, we give some conditions for the measure to be absolutely continuous. Finally, we consider the statistics of these random products and, in joint work with R. Sharp, their extensions to group actions.

Schmidt, Klaus (University of Vienna, Austria & MPI, Germany)

Mahler measure, a dynamical number

This talk is about connections between orbit counts, entropy and Mahler measure for toral automorphisms and much more general algebraic actions of countable discrete groups. Expansiveness appears to play an important role in these connections, and many questions remain open in the absence of expansiveness.

Shapira, Uri (Hebrew University of Jerusalem, Israel & MPIM, Germany)

Applications of rigidity of higher rank hyperbolic actions

In this talk I will describe a general framework linking between dynamics on homogeneous spaces and Diophantine approximations and survey some recent results. In particular I will describe two families of problems: 1) Generalized Littlewood conjectures (GLC); 2) Inhomogeneous Minima problems (IM). Regarding problem (1) I will show that there are cases where GLC holds. Regarding problem (2) I will compute the IM in certain cases.

Applications of the results to Diophantine app' will be given and open problems will be presented.

Skau, Christian (Norwegian U. Scie. and Technology & MPI, Germany)
Orbit equivalence and (ordered) K-theory

To a dynamical system -- either in the measure-theoretic or in the topological setting -- one can associate an operator algebra. The invariants for operator algebras are of (ordered) K-theoretic nature,and this reflects back on the dynamical systems as invariants for orbit equivalence. We will give a survey of this aspect of studying minimal dynamical systems,and we will present some recent results.

Snoha, Lubomir (Matej Bel University, Slovakia & MPI, Germany)
Topology of minimal sets

Discrete dynamical systems given by a continuous map on a topological (usually compact metrizable) space will be considered. Minimality of such a system/map can be defined as the density of all forward orbits. Every compact system contains at least one minimal set, i.e., a nonempty closed invariant subset such that the restriction of the map to this subset is minimal. A fundamental question in topological dynamics is the one on the topological structure of minimal sets of continuous maps in a given space (and, in particular, whether the space itself admits a minimal map or not). In the first part of the talk we will present a survey of some known facts on minimality (including those on topological properties of minimal maps and on noninvertible minimal maps). Then we will concentrate on the topological structure of minimal sets. Among others, recent results obtained jointly with Kolyada and Trofimchuk and with Balibrea, Downarowicz, Hric and Spitalsky will be presented.

Solomyak, Boris (University of Washington, US & MPI, Germany)
Dynamics of self-affine tilings and algebraic number theory

We consider a self-affine tiling (with a diagonalizable expansion matrix) in the Euclidean space and the associated dynamical system, namely, the translation action on the orbit closure of the given tiling. We investigate the spectral properties of the system. It turns out that the presence of the discrete component depends on the algebraic properties of the eigenvalues of the expanding matrix for the tiling. In particular, we prove that the system is weakly mixing if and only if the matrix is totally non-Pisot. The proof is based on a kind of "rigidity" for self-affine tilings and uses some techniques of Thurston and Kenyon. This is a joint work with Jeong-Yup Lee.

Vershik, Anatoly (St. Petersburg Dep.Steklov In-te Mathematics, Russia)
Arithmetic and dynamics

We will discuss the link between the ergodic theory and reordering of the integers. In the general part we will discuss how to retell dynamics in terms of infinite permutations of the naturals. The concrete part of the talk will consist in the discussion on well-known Morse transformation from that point of view. We define a new partial order on the 2-adic numbers $\mathbb{Z}^2$ which can be extend to additive group of all didadic numbers $\mathbb{Q}^2$ and extend on the upper-triangle group of SL(2, $\mathbb{Q}^2$).

Ugarcovici, Ilie (DePaul University, US & MPI, Germany)
Structure of attractors for (a,b)-continued fractions, II

This talk reports on joint work with Svetlana Katok and will continue the presentation from last Tuesday about a two parameter family of continued fractions and the associated one-dimensional maps suggested for consideration by Don Zagier. There exist sufficient conditions for the natural extension maps to have domains with relatively simple geometric structure (finitely rectangular). We prove that the set of parameters for which the geometric

structure is not finitely rectangular belongs to the boundary of the parameter set, and show how one can construct infinitely many such counter-examples. When the domain of the natural extension map is finitely rectangular, we compute explicitly the absolutely continuous invariant measure of the associated interval map. We also address some possible applications of these continued fractions algorithms to coding geodesics on the modular surface.

Ulcigrai, Corinna (University of Bristol, UK & MPI, Germany)

Absence of mixing in locally Hamiltonian flows on surfaces

We consider a class of area-preserving (locally Hamiltonian) flows on a surface of genus g. We are interested in their ergodic properties, especially mixing: it turns out that the presence/absence of mixing depends on the type of fixed points. We had proved that the presence of centers in a generic such flow is enough to create mixing. Recently we showed that if the flow has only simple saddles, it is generically not mixing, but weakly mixing. The results use the representation of these flows as suspensions over interval exchange transformations and Diophantine-type of conditions on the interval exchange transformation.

Wang, Zhiren (Princeton University, US & HCM, Germany)

A quantitative result on actions of commutative toral automorphisms

Let G be an abelian subgroup of SL(d,$\mathbb{Z}$). When G acts totally irreducibly on T^d and is not virtually-cyclic, Berend proved that every orbit on T^d is either the whole torus or finite. With the additional assumption that G is maximal among the subgroups satisfying Berend's condition, we give an effective statement of this result. This is an analogue of the recent quantitative Furstenberg's theorem concerning the x2, x3 action on the circle by Bourgain-Lindenstrauss-Michel-Venkatesh. If time allows we will also mention progress in the general case without the maximal rank assumption. This is part of my thesis work under the supervision of Prof. Elon Lindenstrauss.

Yoccoz, Jean-Christophe (Collège de France & MPI, Germany)

Affine interval exchange maps

I will report on jointwork with S.Marmi and P.Moussa on affine interval exchange maps. We show that for almost every standard interval exchange map T_0 of genus >1, and all possible slopes except for a codimension 1 subspace, any affine interval exchange map T semi-conjugated to T_0 has wandering intervals: in fact, its non-wandering set has zero Lebesgue measure.

Zieve, Michael (Institute for Advanced Study, US)

Intersections of polynomial orbits, and a dynamical Mordell-Lang conjecture

Pick nonlinear f,g in **C**[x]$, and arbitrary x,y in **C**. I will explain the following result: if the orbits {x, f(x), f(f(x)), ...} and {y, g(y), g(g(y)), ... } have infinite intersection, then f and g have a common iterate. The main ingredients in the proof are Siegel's theorem on integral points on curves, a specialization argument due to Lang, a result of Bilu and Tichy on Diophantine equations with infinitely many solutions, and a new description of the set of expressions of a given polynomial as the functional composition of other polynomials. I will also discuss a general question which includes both this result and the Mordell-Lang conjecture.