

SIMILAR RELATIVELY HYPERBOLIC ACTIONS OF A GROUP

LEONID POTYAGAILO

Max-Planck-Institut für Mathematik
Université de Lille 1, France

ABSTRACT. This is a joint work with Victor Gerasimov (University of Belo Horizonte, Brasil).

Let a discrete group G possess two convergence actions by homeomorphisms on compacta X and Y . Consider the following question: does there exist a convergence action of G on a compactum Z and continuous equivariant maps $X \leftarrow Z \rightarrow Y$? We call the space Z (and action of G on it) *pullback* space (action). In such general setting a negative answer follows from a recent result of O. Baker and T. Riley.

Suppose, in addition, that the initial actions are relatively hyperbolic that is they are non-parabolic and the induced action on the space of distinct pairs of points is cocompact. In the case when G is finitely generated the universal pullback space exists by a theorem of V. Gerasimov.

We show that the situation drastically changes already in the case of countable non-finitely generated groups. We provide an example of two relatively hyperbolic actions of the free group G of countable rank for which the pullback action does not exist.

Our main result is that the pullback space exists for two relatively hyperbolic actions of any group G if and only if the maximal parabolic subgroups of one of the actions are dynamically quasiconvex for the other one.

We study an analog of the geodesic flow for a large subclass of convergence groups including the relatively hyperbolic ones. The obtained results imply the main result and seem to have an independent interest.