

## On Curves with Prescribed Number of Points

One of the main problem in curves over finite fields is to determine for which triple  $(q, g, N)$  there exists a curve  $\mathcal{C}$  over the finite field  $\mathbb{F}_q$  of genus  $g$  having exactly  $N$  rational points. A complete solution to the problem is out of reach. Elkies et al. gave a partial answer to the problem and proved that there exists a constant  $\gamma_q > 0$  depending only on  $q$  such that for any integer  $g \geq 0$  there exists a curve  $\mathcal{C}$  over  $\mathbb{F}_q$  of genus  $g$  having  $N \geq \gamma_q g$  rational points. We extend the result of Elkies et al. substantially and proved the following statement:

there are constants  $\alpha_q, \beta_q > 0$  depending only on  $q$  such that for any integer  $g \geq 0$  and non-negative integer  $N \in [0, \alpha_q g - \beta_q]$  there exists a curve  $\mathcal{C}$  over  $\mathbb{F}_q$  of genus  $g$  having exactly  $N$  rational points.

This is a joint work with Henning Stichtenoth.