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Abstract

Computing the Teichmüller polynomial.

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A fibered hyperbolic 3-manifold M is a rich source[1] for pseudo-Anosov mapping classes. Indeed, Thurston's theory of fibered faces tells us that integer points in the *fibered cone* $\mathbb{R}^+ \cdot F \subset H^1(M,\mathbb{R})$ over the *fibered face* F of the Thurston norm unit ball correspond to fibrations of M over the circle. Given that M is hyperbolic, the monodromy of each such fibration is a pseudo-Anosov class $[\psi]$ with stretch factor $\lambda(\psi) > 1$. These stretch factors are packaged in the Teichmüller polynomial, defined by McMullen at the end of the 1990's. This is an element $\Theta_F = \sum_{g\in G} a_g g$ in the group ring $\mathbb{Z}[H_1(M,\mathbb{Z})/\text{Torsion}]$, which is associated to the fibered face F and that is used to compute effectively the stretch factor $\lambda(\psi)$. More precisely, if $[\alpha] \in H^1(M,\mathbb{Z})$ is the integer class in the fibered cone which corresponds to ψ and $\xi_\alpha \in H_1(M,\mathbb{Z})$ is its dual, then the largest root of the Laurent polynomial $\Theta_F(\alpha) := \sum_{g\in G} a_g \cdot t^{\,\xi_\alpha(g)} \in \mathbb{Z}[t,t^{-1}]$ is the stretch factor $\lambda(\psi)$. The Teichmüller polynomial plays a useful role in the construction of mapping classes with small entropy (equivalently small stretch factor). It has been used as a natural source of pseudo-Anosov homeomorphism having small *normalized stretch factors*: infinite families of pseudo-Anosov homeomorphism $[\psi] \in \text{Mod}(\Sigma_g)$ satisfying $\lambda(\psi)^g = O(1)$ as $g \to \infty$.
The main goal of this talk is to present an algorithm to compute explicitly the Teichmüller polynomial and to give a unified presentation of the aforementioned papers. More precisely we will denote the mapping torus of $[\psi] \in \text{Mod}(S)$ by

$$M_\psi := S \times [0,1]/(x,1) \sim (\psi(x),0)$$

and we will suppose that the first Betti number of M_ψ is at least 2.

Based on the results of Penner and Papadopoulos on train tracks and elementary operations (folding operations in the present paper), we provide an algorithm that

(1) computes the Teichmüller polynomial Θ_F of the fibered face F of M_ψ where $[\psi] \in \text{Mod}(\mathbf{D}_n)$ is a pseudo-Anosov class on an n-punctured disc $\mathbf{D}_n$.
(2) computes the topology (genus, number of singularities and type) of the fibers of fibrations in the cone $\mathbb{R}^+ \cdot F$.

As a byproduct, our algorithm allows us also to derive all the relevant informations on the topology of the different fibers that belong to the fibered face.

[1]provided that the first Betti number $b_1(M) \geq 2$.