Selberg and Ruelle zeta functions on compact hyperbolic odd dimensional manifolds

Abstract

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In this talk, we present some recent results concerning the Selberg and Ruelle zeta functions on compact oriented hyperbolic manifolds X of odd dimension d. These are dynamical zeta functions associated with the geodesic flow on the unite sphere bundle S(X). We identify X with $\Gamma\backslash G/K$, where $G = \mathrm{SO}^0(d,1)$, $K = \mathrm{SO}(d)$ and Γ is a discrete torsion-free cocompact subgroup of G. Let $G = KAN$ be the Iwasawa decomposition with respect to K. Let M be the centralizer of A in K. For an irreducible representation σ of M and a finite dimensional representation χ of Γ, we define the Selberg zeta function $Z(s;\sigma,\chi)$ and the Ruelle zeta function $R(s;\sigma,\chi)$. We prove that they converge in some half-plane $\mathrm{Re}(s) > c$ and admit a meromorphic continuation to the whole complex plane. We also describe the singularities of the Selberg zeta function in terms of the discrete spectrum of certain differential operators on X. Furthermore, we provide functional equations relating their values at s with those at $-s$. The main tool that we use is the Selberg trace formula for non-unitary twists. We generalize results of Bunke and Olbrich to the case of non-unitary representations χ of Γ.