Local-global divisibility in commutative algebraic groups

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Abstract

Let k be a number field and let $\mathcal{A}$ be a commutative algebraic group defined over k. We denote by M_k the set of places $v \in k$ and by k_v the completion of k at the valuation v. We consider the following question.

Problem. *Let $P \in \mathcal{A}(k)$. Suppose for all but finitely many $v \in M_k$, there exists $D_v \in \mathcal{A}(k_v)$ such that $P = qD_v$, where q is a positive integer. Is it possible to conclude that there exists $D \in \mathcal{A}(k)$ such that $P = qD$?*

This problem, known as *Local-Global Divisibility Problem*, arises as a generalization of the Hasse principle. The answer is linked to the behaviour of a cohomological group, whose definition is similar to the one of the Tate-Shafarevich group.

We give an overview of the classical answers and the ones given during the last fifteen years for elliptic curves. Furthermore, we show a new result recently obtained in the case of some abelian varieties of higher dimension.