Abstract. The study of affine crystallographic groups has a long history which goes back to Hilbert's 18th problem. More precisely Hilbert (essentially) asked if there is only a finite number, up to conjugacy in $\mathrm{Aff}(\mathbb{R}^n)$, of crystallographic groups Γ acting isometrically on $\mathbb{R}^n$. In a series of papers Bieberbach showed that this was so. The key result is the following famous theorem of Bieberbach. A crystallographic group Γ acting isometrically on the n–dimensional Euclidean space $\mathbb{R}^n$ contains a subgroup of finite index consisting of translations. In particular, such a group Γ is virtually abelian, i.e. Γ contains an abelian subgroup of finite index. In 1964 Auslander proposed the following conjecture

The Auslander Conjecture. *Every crystallographic subgroup Γ of $\mathit{Aff}(\mathbb{R}^n)$ is virtually solvable, i.e. contains a solvable subgroup of finite index.*

In 1977 J. Milnor stated the following question:
Question. *Does there exist a complete affinely flat manifold M such that $\pi_1(M)$ contains a free group ?*

We will explain ideas and methods, recent and old results related to the above problems.