The Globalization Theorem for the Curvature-Dimension Condition, Emanuel Milman.

The Lott–Sturm–Villani Curvature-Dimension condition provides a synthetic notion for a metric-measure space to have Ricci-curvature bounded from below and dimension bounded from above. We prove that it is enough to verify this condition locally: an essentially non-branching metric-measure space $(X, \mathsf{d}, \mathfrak{m})$ (so that $(\mathrm{supp}(\mathfrak{m}), \mathsf{d})$ is a length-space and $\mathfrak{m}(X) < \infty$) verifying the local Curvature-Dimension condition $\mathsf{CD}_{loc}(K, N)$ with parameters $K \in \mathbb{R}$ and $N \in (1, \infty)$, also verifies the global Curvature-Dimension condition $\mathsf{CD}(K, N)$. In other words, the Curvature-Dimension condition enjoys the local-to-global property, rendering all other synthetic notions of Curvature-Dimension such as CD^* and CD^e equivalent to CD in the above setting.

In the talk, we will try to sketch (at least some of) the main new ingredients of the proof: an explicit *change-of-variables* formula for densities of Wasserstein geodesics depending on a second-order temporal derivative of associated interpolating Kantorovich potentials; a surprising *third-order* bound on the latter Kantorovich potentials, which holds in complete generality on any proper geodesic space; and a certain *rigidity* property of the change-of-variables formula, allowing us to bootstrap the a-priori available regularity. The change-of-variables formula is obtained via a new synthetic notion of Curvature-Dimension we dub $\mathsf{CD}^1(K, N)$.

This is joint work with Fabio Cavalletti.