Deformations of the Killing spinor equation on Sasaki-Einstein and 3-Sasaki manifolds

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A simply connected Sasaki-Einstein manifold (M, g) has 2 linearly independent Killing spinors while a 3-Sasaki manifold of dimension $4m-1$ has $m+1$ linearly independent Killing spinors. Infinitesimal transversal holomorphic deformations of the Reeb foliation $\mathscr{F}_\xi$, elements of $H^1_{\bar\partial_b}(\Theta_{\mathscr{F}_\xi})$, where $\Theta_{\mathscr{F}_\xi}$ is the sheaf of transversely holomorphic vector fields, give rise to infinitesimal Einstein deformations of g. We show that under these infinitesimal Einstein deformations the 2 Killing spinors in the first case are preserved to first order, while the remaining $m-1$ in the 3-Sasaki case are never preserved.

In the Sasaki-Einstein case the integrability of these infinitesimal deformations is unanswerable in general, but the elements of $H^1_{\bar\partial_b}(\Theta_{\mathscr{F}_\xi})^T$, with $T \subset \mathrm{Aut}(M, g)$ a maximal torus, do give deformations. Furthermore, one can prove that in the 3-Sasaki case the real subspace $\mathrm{Re}\, H^1_{\bar\partial_b}(\Theta_{\mathscr{F}_\xi})$ integrates to actual Sasaki-Einstein deformations, i.e. preserving precisely a 2 dimensional subspace of Killing spinors. We consider this in the case of toric 3-Sasaki 7-manifolds. The underlying Sasaki-Einstein structure of a toric 3-Sasaki 7-manifold M has a real $b_2(M)-1$ dimensional space of Sasaki-Einstein deformations, where $b_2(M)$ can be arbitrarily large. Thus we have examples of Einstein 7-manifolds admitting 3 Killing spinors which have a family of deformations to Einstein metrics admitting only 2 Killing spinors. Therefore, unlike the case of parallel spinors, the dimension of the space of Killing spinors does not always remain constant under Einstein deformations.