

Automorphism-invariant random subspaces of a locally compact vector space over a finite field, and IRS in hyperbolic and mapping class groups

We describe the notion and applications of invariant random subgroup (IRS) which has recently gained a lot of attention. An IRS of a group G is a probability measure on the space of all closed subgroups of G , which is conjugation invariant. It is a probabilistic version of normal subgroup. Any measure preserving action of G produces an IRS by considering stabilizers of points, and conversely, any IRS comes from a measure preserving action.

The notion of IRS was used by M. Abért et al. [1] to study asymptotic behavior of Betti numbers and other spectral invariants of sequences of locally symmetric spaces. L. Bowen [2] has used it to provide examples of actions with quasi-invariant measure which have arbitrary Furstenberg entropy.

It is interesting to study the set of all IRS's of a given group. L. Bowen [3] has shown that there is a Poulsen subsimplex in the set of all IRS's of a free non-abelian group. Together with R. Grigorchuk and L. Bowen [4] we have shown that the set of all IRS's of the classical Lamplighter group also contains a Poulsen subsimplex.

Together with R. Grigorchuk and L. Bowen [5] we have developed a notion of characteristic random subgroup, CRS. It is a measure on the set of subgroup, invariant with respect to all automorphisms of a group G , and thus it is a probabilistic counterpart of the notion of characteristic subgroup. We have described the simplex of all CRS's of a free abelian group of infinite rank and of infinite dimensional torus. This has allowed to construct atomless IRS's of groups of geometric nature, in particular in hyperbolic and mapping class groups.

References

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- [2] L. Bowen, *Random walks on random coset spaces with applications to Furstenberg entropy*. Invent. Math. 196 (2014), no. 2, 485–510.

- [3] L. Bowen, *Invariant random subgroups of the free group*. arXiv:1204.5939.
- [4] L. Bowen, R. Grigorchuk, R. Kravchenko, *Invariant random subgroups of the lamplighter group*. arXiv:1206.6780.
- [5] L. Bowen, R. Grigorchuk, R. Kravchenko, *Characteristic random subgroups of geometric groups and free abelian groups of infinite rank*. arXiv:1402.3705.