

# Conference for Young researchers in homotopy theory and categorical structures

GONG SHOW - February 13, 2017

## Titles and abstracts

HONGYI CHU (LILLE 1 UNIVERSITY)

### Rectification Results and Enriched dendroidal Segal Spaces

In the classical theory of operads, enriched operads can be viewed as algebraic objects. In this talk I will first show how this idea leads to the definition of enriched infinity operads. Then I will introduce an enriched version of the concept of dendroidal Segal spaces. At the end of the talk the theory of operadic approximations developed by Lurie is used to show that enriched dendroidal Segal spaces are enriched infinity operads. In particular, this result provides another proof for the equivalence between simplicial operads and dendroidal Segal spaces, which was first proven by Cisinski and Moerdijk.

ALICE HEDENLUND (UNIVERSITY OF OSLO)

### The Tate spectral sequence for compact Lie groups

The Tate spectral sequence is a spectral sequence that calculates the homotopy groups of the Tate construction of a  $G$ -spectrum for a finite group  $G$ . This is a short expository talk on the subject where I sketch how Greenlees-May and Hesselholt-Madsen construct the Tate spectral sequence for finite groups, and give a possible generalization to compact Lie groups.

FOSCO LOREGIAN (UNIVERSITY OF WESTERN ONTARIO)

### t-derivators

I will present a (very sketchy) work in progress with S. Virili (UAB) and F. Mattiello (Università degli studi di Padova). In a stable infinity-category  $t$ -structures correspond bijectively to certain factorization systems [Lor16, the "Rosetta stone" theorem]; we try to export the same result to the theory of derivators, where a "calculus of factorization systems" seems a pretty elusive concept to define (as well as a satisfying notion of  $t$ -structure).

We start noticing that a sufficient condition for a 2-category to "express the theory of factorization systems" is to have a monad  $C \mapsto C^2$  whose algebras are precisely certain objects endowed with a (one-sided) inverse of the composition functor [KT]. This describes "formally" the theory of factorizations, and allows to speak about an object  $C$  of a 2-category  $K$  endowed with a factorization system.

We exploit the "formal theory of monads" [Lag16] in the category  $(P)Der$  to extend the validity of [Lor16]'s "Rosetta stone" for stable derivators, and we sketch a foundation for the theory of  $t$ -structure thereof.

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[KT]: Korostenski, Mareli, and Walter Tholen. "Factorization systems as Eilenberg-Moore algebras." *Journal of Pure and Applied Algebra* 85.1 (1993): 57-72. [Lag16]: Lagkas-Nikolos, Ioannis. "Levelwise modules over separable monads on stable derivators." *arXiv preprint arXiv:1608.06340* (2016). [Lor16]: Fiorenza, Domenico, and Fosco Loregian. "t-structures are normal torsion theories." *Applied Categorical Structures* 24.2 (2016): 181-208.

JOSÉ MANUEL MORENO-FERNANDEZ (UNIVERSIDAD DE MÁLAGA)

### Higher order Whitehead products and $L_\infty$ structures on the homology of a dg Lie algebra

We will study the inherited  $L_\infty$  structure on the homology of a DGL and its relationship with the higher order rational Whitehead products in Quillen's models for rational homotopy theory [4].

More specifically, if we denote by  $H$  the homology of a dg Lie model of a space  $X$  (so that  $H \cong \pi_*(\Omega X) \otimes \mathbb{Q}$  as graded Lie algebras), we will sketch how and when one can detect higher order Whitehead products of order  $k$  (which are homotopy sets arising from a natural extension problem [3]) via the higher operation of order  $k$  induced on  $H$  via the Homotopy Transfer Theorem (equivalently, by Homological Perturbation Theory [2]).

The talk is based on joint work with F. Belchi, U. Buijs and A. Murillo [1].

#### References

- [1] F. Belchi, U. Buijs, J. M. Moreno-Fernandez, A. Murillo, Higher order Whitehead products and  $L_\infty$  structures on the homology of a DGL, arXiv:1604.01478, (2016).
- [2] J. Huebschmann, J. Stasheff, Formal solution of the master equation via HPT and deformation theory, *Forum Math.* **14** (2002), 847-868.
- [3] G. J. Porter, Higher order Whitehead products, *Topology* **3** (1965), 123–135.
- [4] D. Quillen, Rational homotopy theory, *Ann. of Math. (2)* **90** (1969), 205–295.

STEPHEN NAND-LAL (THE UNIVERSITY OF LIVERPOOL)

### A Simplicial Approach to the Homotopy Theory of Stratified Spaces

In this talk I will introduce a transferred model category on stratified spaces, allowing a categorical approach to studying the homotopy theory of stratified spaces. Initially we construct an adjunction between our category of stratified spaces and simplicial sets, arising in the same manner as the adjunction between geometric realisation and singular simplicial set functor. This follows from the general theory of adjunctions with simplicial sets.

The stratified adjunction allows cofibrant transfer of the Joyal model structure for quasi-categories to the category of stratified spaces, inherently giving a close relationship between the fibrant stratified spaces and quasi-categories. Importantly this allows theory of quasi-categories to be invoked, proving a myriad of results for the fibrant stratified spaces. Upon further analysing the transferred model structure, it can be shown that the cofibrant-fibrant stratified spaces satisfy the properties of homotopically stratified sets, a well established setting for studying the homotopy theory of stratified spaces.

There are a number of reasons to believe this is the correct setting for studying stratified homotopy theory; for example, it follows from Whitehead's Theorem that a weak equivalence between cofibrant-fibrant objects is a stratum preserving homotopy equivalence. In addition, interpreting the Joyal characterisation of a weak equivalence between quasi-categories enables the characterisation of a stratum preserving homotopy equivalences between cofibrant-fibrant stratified spaces as those stratified morphisms which induce a bijection between posets, and induces weak homotopy equivalences between corresponding homotopy links (and hence between strata).

This is a joint work with my supervisor, Jon Woolf.

MARCEL RUBIÓ (KU LEUVEN)

### Cohomology jump functors via $L_\infty$ -pairs

To a homotopy algebra (or operad), one may associate its deformation theory via dglas. It is known [2] that two  $\infty$ -quasi-isomorphic homotopy algebras  $\mathcal{A}$  and  $\mathcal{B}$  will have quasi-isomorphic deformation complexes  $\text{Def}(\mathcal{A})$  and  $\text{Def}(\mathcal{B})$  as  $L_\infty$ -algebras. That is, the deformation complexes are homotopy invariants of homotopy algebras.

Budur and Wang [1] refined this homotopy invariant by introducing cohomology jump deformation functors via dgla-pairs. In this case, the homotopy invariance of the homotopy algebra is detected as quasi-isomorphisms of  $L_\infty$ -pairs instead. We will discuss what does formality mean in this setting, how to deal with non-formal dgla-pairs and why is this refinement interesting.

#### References

- [1] Nero Budur and Botong Wang. Cohomology jump loci of differential graded Lie algebras. *Compos. Math.*, 151(8):1499-1528, 2015
- [2] Vasily Dolgushev and Thomas Willwacher. The deformation complex is a homotopy invariant of a homotopy algebra. In *Developments and retrospectives in Lie theory*, volume 38 of *Dev. Math.*, pages 137-158. Springer, Cham, 2014.

STEPHANIE ZIEGENHAGEN (KTH ROYAL INSTITUTE OF TECHNOLOGY)

### **Topological coHochschild homology**

Topological coHochschild homology ( $\mathrm{coTHH}$ ), recently defined by Kathryn Hess and Brooke Shipley, is an invariant of coalgebra spectra dual to topological Hochschild homology ( $\mathrm{THH}$ ). For suspension spectra  $\mathrm{coTHH}$  is closely related to  $\mathrm{THH}$  and string topology.

In this talk, I will discuss a Bökstedt type spectral sequence for  $\mathrm{coTHH}$  and explain how to equip this spectral sequence with a coalgebra structure. Hochschild homology of free commutative dg-algebras can be explicitly identified using the Hochschild-Kostant-Rosenberg theorem. An analogous result for coHochschild homology together with the coalgebra structure on the spectral sequence allows us to compute the mod  $p$  homology of  $\mathrm{coTHH}$  in certain cases. This is joint work with Anna Marie Bohmann, Teena Gerhardt, Amalie Høgenhaven and Brooke Shipley.