MARKOV TYPE CONSTANTS OF WASSERSTEIN SPACES

For $p > 1$, $T \in \mathbb{N}$ and a metric space X we debote by $M_p(X,T)$ the Markov type p constant at time T of X. The Markov type p constant of X, denoted by $M_p(X)$ is defined by

$$M_p(X) = \sup_{T\in\mathbb{N}} M(X,T) \in [1,\infty].$$

We say that X has Markov type p if $M_p(X) < \infty$. The notion of Markov type was introduced by K. Ball [1] and has two main applications:

(1) Partial Lipshitz maps from a metric space X having Markov type p are extendable to Lipshitz maps of the whole space.
(2) Obstruction to bi-Lipshitz embeddability. Metric space X can not be embedded into metric space Y with a bi-Lipshitz distortion less then $\sqrt{\frac{M_p(X,T)}{M_p(Y,T)}}$.

In the talk I'm going to present an idea which allows to compute Markov type constants for Wasserstein spaces. Let $\mathscr{P}_p(\mathbb{R}^d)$ denotes the Wasserstein space over the Euclidean space $\mathbb{R}^d$. We obtain following estimates.

Theorem ([3], Corollary 3)**.** *For every $p \in (2,\infty)$ and $T, d \in \mathbb{N}$ we have*

(1) $M_p(\mathscr{P}_p(\mathbb{R}^d), T) \leq 16 d^{1/2-1/p} p^{1/2} T^{1/2-1/p}$,
(2) $M_2(\mathscr{P}_p(\mathbb{R}^d)) \leq 4 d^{1/2-1/p}\sqrt{p-1}$.

As observed by A. Andoni, A. Naor and O. Neiman the upper bound for $M_p(\mathscr{P}_p(\mathbb{R}^d), T)$ implies certain restriction on embeddability of snowflakes into $\mathscr{P}_p(\mathbb{R}^d)$. Theorem(2) provides an extension theorem for partial Lipshitz maps from $\mathscr{P}_p(\mathbb{R}^d)$ into CAT(0) spaces, uniformly convex Banach spaces or more generally metric spaces with metric Markov cotype 2(See [2, Theorem 1.11, Corollary 1.13]).

References

[1] K. Ball. Markov chains, Riesz transforms and Lipschitz maps. *Geometric and Functional Analysis GAFA*, 2(2):137–172, 1992.

[2] Manor Mendel and Assaf Naor. Spectral calculus and lipschitz extension for barycentric metric spaces. *Analysis and Geometry in Metric Spaces*, 1:163–199.

[3] Vladimir Zolotov. Markov type constants, flat tori and wasserstein spaces. *arXiv preprint arXiv:1610.04886*, 2016.