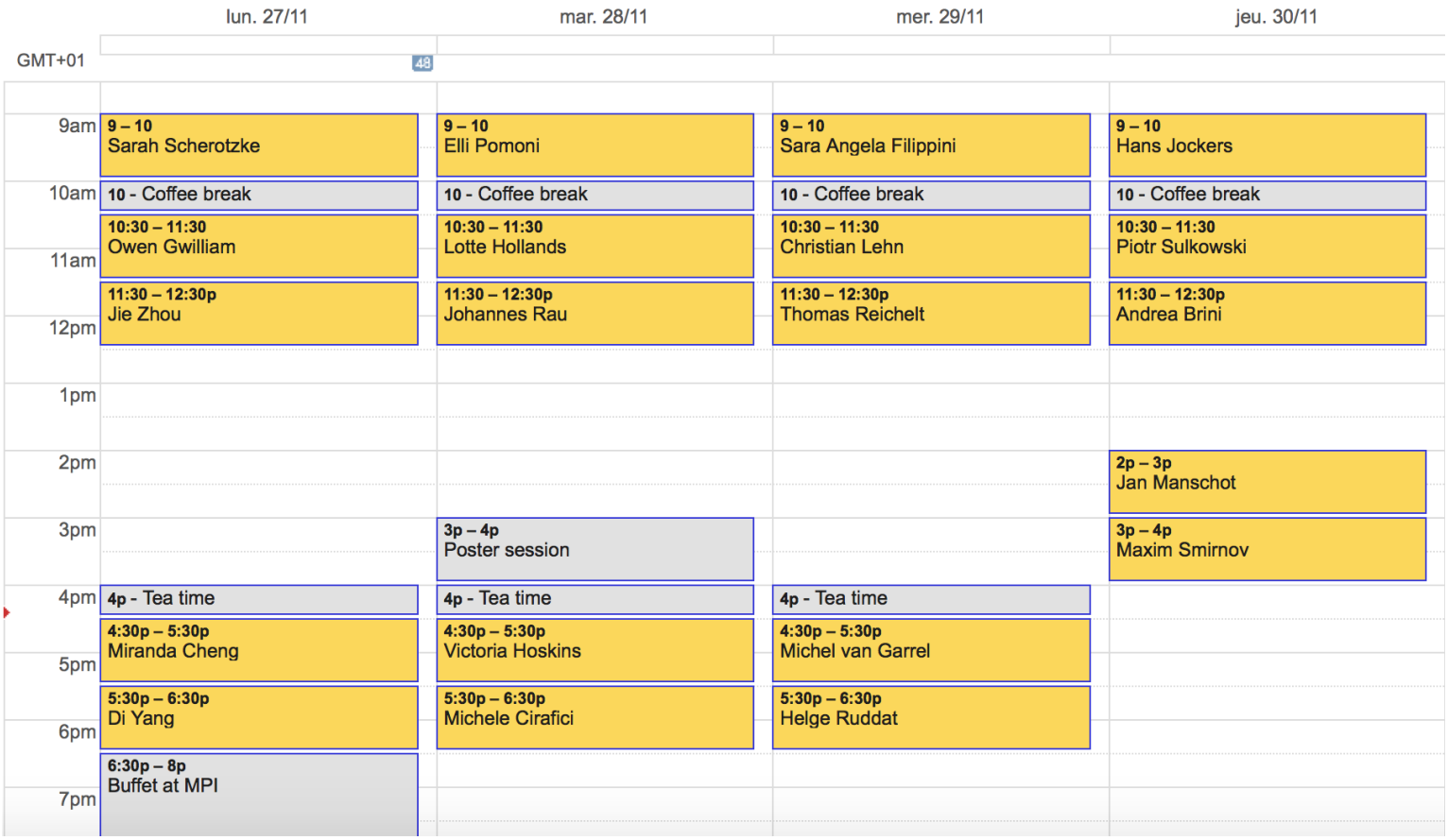

Workshop, November 27th-30th, 2017

Young researchers in string mathematics

Talks

◦ Andrea Brini (CNRS Montpellier/Imperial College)
Non-perturbative spectra, quantum curves and mirror symmetry

"Quantum curves" have been all the rage for various subsectors of the geometry/mathematical physics community for the last few years; yet they might mean different things for different people. I will focus on one and only one angle of this story, due to Grassi–Hatsuda–Kashaev–Mariño–Zakany: in their setting, "quantum curves" is the monicker of a precise connection between the spectral theory of a class of difference operators on the real line with trace-class resolvent, and the enumerative geometry (GW/DT invariants) of toric threefolds with trivial canonical bundle. That's both bizarre – the two subjects have a priori little overlap, and share few-to-none joint practitioners – and deep/beautiful/powerful to various degrees. I will first review part of the existing depth/beauty/power of this story, and then outline some work in progress aimed at applying mirror-symmetry techniques (in particular, the theory of mutations of potentials of Galkin–Usnich) to the computation of quantum mechanical spectra.

◦ Miranda Cheng (University of Amsterdam)
TBA

◦ Michele Cirafici (IST Lisboa)
Line defects in $\mathcal{N} = 2$ QFT and framed quivers

I will discuss a certain class of line defects in four dimensional supersymmetric theories with $\mathcal{N} = 2$. Many properties of these operators can be rephrased in terms of quiver representation theory. In particular one can study BPS invariants of a new kind, the so-called framed BPS states, which correspond to bound states of ordinary BPS states with the defect. Such invariants determine the IR vev of line operators. I will discuss how these invariants arise from framed quivers. Time permitting I will also discuss a formalism to study these quantities based on cluster algebras.

◦ Sara Angela Filippini (Cambridge University)
Stability data, irregular connections and tropical curves

I will outline the construction of isomonodromic families of irregular meromorphic connections on $\mathbb{P}^1$ with values in the derivations of a class of infinite-dimensional Poisson algebras, and describe two of their scaling limits. In the "conformal limit" we recover a version of the connections introduced by Bridgeland and Toledano-Laredo, while in the "large complex structure limit" the connections relate to tropical curves in the plane and, through work of Gross, Pandharipande and Siebert, to tropical/GW invariants. This is joint work with M. Garcia-Fernandez and J. Stoppa.

◦ Michel van Garrel (Universität Hamburg)
Comparing local and log GW invariants

Let X be a smooth projective variety and let D be a smooth nef divisor on it. In this collaboration with Tom Graber and Helge Ruddat, we show that the genus 0 local Gromov-Witten (GW) invariants of the total space of $\mathcal{O}(-D)$ equal, up to a factor, the genus 0 log GW invariants of X with a single condition of maximal contact order along D.

◦ Owen Gwilliam (MPIM)
Large N limits from a BV perspective

Starting with 't Hooft, physicists have used a ribbon graph expansion to understand certain integrals over spaces of $N \times N$ matrices in the large N limit. This expansion can be deduced from the Feynman diagram expansion, which relies on the nice structure of moments of a Gaussian measure. We provide a homological perspective on this situation: the Batalin-Vilkovisky formalism (which we will outline) provides a homological approach to computing moments, and the Loday-Quillen-Tsygan theorem (which we will explain) gives a method for identifying the large N limit. Our talk will focus on finite-dimensional integrals, but we hope to comment on how our methods extend to gauge theories and connect with string field theory. This is joint work with Greg Ginot and Mahmoud Zeinalian.

∘ Lotte Hollands (Heriot-Watt University, Edimburgh)
Geometric recipe for superpotentials

Nekrasov, Rosly and Shatashvili observed that the generating function of a certain space of $\mathrm{SL}(2)$ opers has a physical interpretation as the effective twisted superpotential for a four-dimensional $\mathcal{N} = 2$ quantum field theory. In this talk we describe the ingredients needed to generalise this observation to higher rank. Important ingredients are spectral networks generated by Strebel differentials and the abelianization method. As an example we find the twisted superpotential for the E_6 Minahan-Nemeschansky theory.

∘ Victoria Hoskins (Humboldt Universität)
Group actions on quiver varieties and application to branes

We study two types of actions on King's moduli spaces of quiver representations over a field k, and we decompose their fixed loci using group cohomology in order to give modular interpretations of the components. The first type of action arises by considering finite groups of quiver automorphisms. The second is the absolute Galois group of a perfect field k acting on the points of this quiver moduli space valued in an algebraic closure of k; the fixed locus is the set of k-rational points, which we decompose using the Brauer group of k, and we describe the rational points as quiver representations over central division algebras over k. Over the field of complex numbers, we describe the symplectic and holomorphic geometry of these fixed loci in hyperkähler quiver varieties using the language of branes. This is joint work with Florent Schaffhauser.

∘ Hans Jockers (BCTP Bonn)
Mirror symmetry of branes and hyperbolic 3-manifolds

We discuss the computation of normal functions between the van Geemen lines on the mirror quintic Calabi-Yau threefold in a certain semi-stable degeneration limit. In this limit the normal functions are described as elements of higher Chow groups. Physically this amounts to computing the domain wall tension between certain B-branes on the mirror quintic in the large complex structure limit. By mirror symmetry we expect that these normal functions/domain wall tensions have a geometric meaning on the quintic Calabi-Yau threefold for suitable A-branes. As we discuss, the number theoretic structure of the computed normal functions indicates that the relevant A-branes correspond to hyperbolic 3-manifolds.

∘ Christian Lehn (Universität Chemnitz)
Birational geometry of singular symplectic varieties and a global Torelli theorem

Verbitsky's global Torelli theorem has been one of the most important advances in the theory of holomorphic symplectic manifolds in the last years. In a joint work with Ben Bakker (University of Georgia) we prove a version of the global Torelli theorem for singular

symplectic varieties and discuss applications. Symplectic varieties have interesting geometric as well as arithmetic properties, their birational geometry is particularly rich. We focus on birational contractions of symplectic varieties and generalize a number of known results for moduli spaces of sheaves to general deformations. Our results are obtained through the interplay of Hodge theory, deformation theory, and a further example of Verbitsky's technique which might carry the name "how to deduce beautiful consequences from ugly behavior of moduli spaces".

◦ Jan Manschot (Trinity College Dublin)
Topological partition functions and (iterated) integrals of modular forms

As a consequence of electric-magnetic duality, partition functions of four-dimensional gauge theories can be expressed in terms of modular forms in many cases. I will discuss new results for the modularity of topologically twisted partition functions of N=2 and N=4 supersymmetric theories, and in particular how these partititon functions may involve (iterated) integrals of modular forms.

◦ Elli Pomoni (DESY Hamburg)
Exact results for class S_k

We will introduce a large class of $\mathcal{N} = 1$ superconformal theories, called S_k, which is obtained from Gaiotto's $\mathcal{N} = 2$ class S via orbifolding. We will study the Coulomb branch of the theories in the class by constructing and analyzing their spectral curves. Using our experience from the $\mathcal{N} = 2$ AGT correspondence we will search for a 2d/4d relations (AGT_k) for the $\mathcal{N} = 1$ theories of class S_k. From the curves we will identify the 2d CFT symmetry algebra and its representations, namely the conformal blocks of the Virasoro/W-algebra, that underlie the 2d theory and reproduce the Seiberg-Witten curves of the $\mathcal{N} = 1$ gauge theories. We find that the blocks corresponding to the $\text{SU}(N)$ S_k gauge theories involve fields in certain non-unitary representations of the $W_{k,N}$ algebra. These conformal blocks give a prediction for the instanton partition functions of the 4d $\mathcal{N} = 1$ SCFTs of class S_k.

◦ Johannes Rau (Universität Tübingen)
Tropical Hurwitz and GW numbers

Tropical geometry has been proved successful to study various types of enumerative numbers, including Gromov-Witten invariants for toric surfaces and Hurwitz numbers with at most two special points. In my talk I will try to give an overview on some showcase results, recent developments (counting "real" curves) and relations to other approaches.

◦ Thomas Reichelt (Universität Heidelberg)
Global mirror symmetry

Conjecturally, global mirror symmetry connects the quantum cohomology of projective varieties which are birational. In this talk, I will focus on the simplest case of a (dis-)crepant blow-up and explain the construction of the corresponding global Landau-Ginzburg model

◦ Helge Ruddat (Universität Mainz)
Analyticity of Gross–Siebert Calabi–Yau families

Gross and Siebert gave an algorithm to produce from toric degeneration data a canonical formal Calabi–Yau family. Siebert and I prove that this family is in fact the completion of an analytic family. In particular, its nearby fibres are decent Calabi-Yau manifolds over the complex numbers. Furthermore, the family is semi-universal, *i.e.* is in a sense locally the

moduli space of Calabi–Yaus. The key result on the route to analyticity is the computation of canonical coordinates on the base by explicit integration of a holomorphic volume form over topological cycles that we construct from tropical 1-cycles in the base of the SYZ-fibration.

∘ Sarah Scherotzke (Universität Münster)
Categorification of Chern characters.

The Chern character is a central construction with incarnations in algebraic topology, representation theory and algebraic geometry. It is an important tool to probe K-theory, which is notoriously hard to compute. In my talk, I will explain, what the categorification of the Chern character is and how we can use it to show that certain classical constructions in algebraic geometry are of non-commutative origin. The category of motives plays the role of K-theory in the categorified picture. The categorification leads also to the construction of higher invariants such as the secondary Chern characters and secondary K-theory.

∘ Maxim Smirnov (Universität Augsburg/MPIM Bonn)
On Lefschetz exceptional collections and quantum cohomology of Grassmannians

Given a Lefschetz exceptional collection on a variety X one defines its residual subcategory as the orthogonal to the rectangular part of the collection. In this talk we will discuss some conjectural relations between the quantum cohomology of X and the structure of the residual subcategory motivated by homological mirror symmetry. We give examples of this relation when X is an ordinary or a symplectic isotropic Grassmannian.

∘ Piotr Sułkowski (University of Warsaw)
Knots-quivers correspondence

I will present a surprising relation between knot invariants and quiver representation theory, motivated by various string theory constructions involving BPS states. Consequences of this relation include the proof of the famous Labastida-Marino-Ooguri-Vafa conjecture (at least for symmetric representations), explicit (and unknown before) formulas for colored HOMFLY polynomials for various knots, new viewpoint on knot homologies, a novel type of categorification, new dualities between quivers, and many others.

∘ Di Yang (MPIM)
On the Hodge-GUE correspondence

We discuss the recent Hodge–GUE correspondence conjecture on an explicit relationship between special cubic Hodge integrals over the moduli space of stable algebraic curves and enumeration of ribbon graphs with even valencies. We sketch a proof of this conjecture based on the Virasoro constraints. We also discuss the conjectural relationship between the cubic Hodge integrals satisfying the local Calabi–Yau condition and the Bogoyavlensky–Toda hierarchy (*aka* fractional KdV). The talk is based on a series of joint works with B. Dubrovin, S.-Q. Liu and Y. Zhang.

∘ Jie Zhou (Universität Köln)
From classical Weierstraß elliptic functions to quantum invariants

I will talk about a joint work with Si Li on the computation of higher genus B-model for elliptic curves.

I will first formulate the Feynman amplitudes in the higher genus B-model (Kodaira-Spencer theory) in terms of cohomological parings. Then I will discuss properties of the Feynman amplitudes, including the origin of their quasi-modularity, the geometric Inter-

pretation of their modular completions, etc. Finally I explain the implication of the cohomological reformation in renormalization.

Our method mainly uses the basic theory of algebraic curves. It applies to a large class of two-dimensional conformal field theories and can hopefully find application in Eynard-Orantin topological recursion as well.

Posters

◦ Nezhla Aghaei (Bern Universität)
Quantization of Teichmüller theory and super generalization

The quantization of the Teichmüller spaces of Riemann surfaces has found important applications to conformal field theory and $\mathcal{N} = 2$ supersymmetric gauge theories. We construct a quantization of the Teichmüller spaces of super Riemann surfaces, using coordinates associated to the ideal triangulations of super Riemann surfaces. A new feature is the non-trivial dependence on the choice of a spin structure which can be encoded combinatorially in a certain refinement of the ideal triangulation. We construct a projective unitary representation of the groupoid of changes of refined ideal triangulations. Therefore, we demonstrate that the dependence of the resulting quantum theory on the choice of a triangulation is inessential.

In the quantum Teichmüller theory, it was observed that the key object defining the Teichmüller theory has a close relation to the representation theory of the Borel half of $U_q(sl(2))$. In our research we observed that the role of $U_q(sl(2))$ is taken by quantum superalgebra $U_q(osp(1|2))$. A Borel half of $U_q(osp(1|2))$ is the super quantum plane. The canonical element of the Heisenberg double of the quantum super plane is evaluated in certain infinite dimensional representations on $L^2(\mathbb{R}) \otimes \mathbb{C}^{1|1}$ and compared to the flip operator from the Teichmüller theory of super Riemann surfaces.

◦ Florian Beck (Hamburg Universität)
Hitchin system via Calabi-Yau threefolds

Integrable systems are often constructed from geometric and/or Lie- theoretic data. Two important example classes are Hitchin systems and Calabi-Yau integrable systems. A Hitchin system is constructed from a compact Riemann surface together with a complex Lie group with mild extra conditions. In contrast, Calabi–Yau integrable systems are constructed from a priori purely geometric data, namely certain families of Calabi–Yau threefolds. Despite their different origins there is a non-trivial relation between Hitchin and Calabi–Yau integrable systems. More precisely, we have shown that any Hitchin system for a simply-connected or adjoint simple complex Lie group is isomorphic to a Calabi–Yau integrable system (away from singular fibers).

◦ Boulos el-Hilany (Universität Tübingen/MPIM Bonn)
Real Hurwitz numbers and simple rational functions

Real Hurwitz numbers count the number of degree d, genus g ramified covers, of the complex projective line with fixed ramification profiles over their respective branch points. We consider the problem of counting covers having genus zero and endowed with a real structure coming from the complex conjugation, such that all branch points are real. By imposing a particular ramification profile on one branch point, we thus associate to every covering a sign that depends on the position of the real ramification points. Analogously to the works of Itenberg and Zvonkine concerning real polynomial Hurwitz numbers, we prove that the total number of branched coverings, counted with sign, does not depend on the position of the real branch points.

◦ Elba Garcia-Failde (MPIM Bonn)
Simple maps, Hurwitz numbers and topological recursion

In this poster, we call ordinary maps a certain type of graphs embedded on surfaces, in contrast to fully simple maps, which we introduce as maps in which the boundaries do not

touch each other, neither themselves. It is well-known that the generating series of ordinary maps satisfy the topological recursion. For maps whose faces are imposed to be homeomorphic to disks, we prove that after applying the symplectic transformation of exchanging x and y in the spectral curve, the topological recursion enumerates fully simple maps. For more general types of maps, this claim is still a conjecture. We give explicit combinatorial proofs for disks and cylinders, recovering formulas already known in the context of free probability. For the rest of topologies, our proof relies on a matrix model interpretation of fully simple maps, via the formal hermitian matrix model with external field. We also give a universal relation between fully simple and ordinary maps involving double monotone Hurwitz numbers. In particular, we obtain an ELSV-like formula for strictly monotone double Hurwitz numbers with ramification profile $(2, \dots, 2)$ over 0 and arbitrary one over ∞.

○ Andreas Gerhardus (BCTP Bonn)
The geometry of 2*d gauge theories*

Two-dimensional supersymmetric gauge theories provide a powerful tool for studying string compactifications. Recent advances in supersymmetric field theory allow us to non-perturbatively study a certain class of their correlation functions, which geometrically are sections of the quantum moduli space of the string compactification. We relate these to a quadratic expression in the Givental I-function and its derivative, and further present a methodology for the systematic derivation of relations amongst them, which relies on the defining gauge theory data only. In a Hilbert space interpretation these relations correspond to differential operators annihilating the moduli-dependent vacuum ground state. For the case of Calabi-Yau compactifications we thereby obtain the GKZ and Picard-Fuchs differential operators governing their quantum geometry. This enables us to derive universal formulae for Picard-Fuchs operators in terms of gauge theory correlation functions.

○ John Huerta (IST Lisboa)
M-theory from the superpoint

One mysterious facet of M-theory is how a 10-dimensional string theory can "grow an extra dimension" to become 11-dimensional M-theory. In the work of Fiorenza, Sati and Schreiber, this extra dimension arises by the mathematical process of central extension. Moreover, all the strings and branes of string theory/M-theory are shown to fit into a purely mathematical pattern described by rational homotopy theory, called the "brane bouquet". We describe work with Schreiber where we show that the brane bouquet arises from the rational homotopy theory of the simplest kind of supergeometry: the superpoint, $\mathbb{R}^{0|1}$.

○ Aknazar Kazhymurat (NIS Almaty)
Lower energy bounds for Lagrangian tori in $\mathbb{C}P^2$

The geometry of Lagrangian tori in $\mathbb{C}P^2$ has well-known links to integrable PDE (Tzizeica equation, 2d Schrödinger equation). Recently, Ma, Mironov and Zuo have introduced the energy functional for Lagrangian tori in $\mathbb{C}P^2$ exploiting these links. The energy functional admits an interesting geometric interpretation, in some sense generalizing the Willmore functional for tori in $\mathbb{R}^3$. A natural analogue of the famous Willmore conjecture (proved by Neves and Marques) in the setting of Lagrangian tori states that the Clifford torus is unique (up to ambient isometries) global minimum of the energy functional. The talk is devoted to some recent progress related to this conjecture

○ Ana Kontrec (University of Zagreb)
Representations of certain non-rational W-algebras

This is a preliminary report. We study representations of certain irrational W-algebras. In particular, we study orbifold subalgebras of Weyl vertex algebras and certain affine W-algebras. In some special cases, we present formulas for singular vectors, classification of irreducible modules and describe the structure of Zhu's algebras.

◦ Danilo Lewański (MPIM Bonn)
Wall-crossing formulae and chamber polynomiality for Hurwitz theory

We derive explicit formulae for the generating series of mixed Grothendieck dessins d?enfant/monotone/simple Hurwitz numbers, via the semi-in nite wedge formalism. This reveals the strong piecewise polynomiality in the sense of Goulden–Jackson–Vakil, generalising a result of Johnson, and provides a new explicit proof of the piecewise polynomiality of the mixed case. Moreover, we derive wall-crossing formulae for the mixed case. These statements specialise to any of the three types of Hurwitz numbers, and to the mixed case of any pair. Based on a joint work with M. Hahn and R. Kramer.

◦ Dmitry Noshchenko (Warsaw University)
Higher Airy structures from Bouchard–Eynard topological recursion

We describe Lie-algebraic structures, attached to spectral curves with higher ramification locally near branch points. The construction is determined by Bouchard–Eynard topological recursion formula, and the resulting algebra of differential operators is studied in details. Two important cases are considered: r-Airy curve $x = y^r$ and its deformation, parametrized by KdV-r times.

◦ Eugene Rabinovich (MPIM Bonn)
The axial anomaly, determinant line, and partition function in the BV formalism

The conventional wisdom about anomalies in fermionic theories is that they measure an obstruction to the construction of a well-defined partition function. We use the BV formalism and the methods of equivariant quantization of Costello and Gwilliam to study the axial anomaly of the free fermion. In particular, we show that the BV formalism provides an interpretation of this conventional wisdom and we compute the result, well-known to physicists, that the axial anomaly of the free fermion is given by the index of the corresponding Dirac operator.

◦ Mohammad Reza Rahmati (UNAM)
Mirror symmetry, zeta functions and Mackey functors

We give a new method to compare the zeta functions of mirror polynomials using theory of Mackey functors. There exists a proof of this fact by Ebeling and Gusein-Zadeh (2014), However our method is quite different and more theoretical than them. Specifically we apply a machinery of Mackey functors on the family of conjugacy classes in G for an orbifold $[X/G]$. We compare these two procedure in a mirror pair of dual invertible polynomials of Bergland–Hubsch in mirror symmetry.

◦ Philipp Rüter (Heriot-Watt University)
Spectral networks and abelianization for wild opers

We will show the construction of special Darboux coordinates on the moduli space of irregular flat $\mathrm{SL}(2)$ connections on a Riemann surface. Our main examples will be a simple class of Argyres-Douglas theories, corresponding to the Riemann sphere with one irregular puncture. The coordinates are used to obtain a generating function of the brane of opers,

which can be identified with the effective twisted superpotential.

◦ Martin Vogrin (Hamburg Universität)
Differential rings for elliptically fibered K_3 surfaces

Special geometry of Calabi-Yau moduli spaces dictates that for every Calabi–Yau manifold there exists an associated finitely generated differential ring. The elements of the differential ring are generalizations of classical modular forms, and reduce to them in special cases. We call a Calabi–Yau variety, together with the associated differential ring an enhanced Calabi–Yau variety, and its moduli space T can be thought of as an extension of the complex structure moduli space by a choice of generators of the differential ring at every point. I will show that for elliptically fibred K_3 surfaces, realised as hypersurfaces in toric ambient spaces, the differential ring is isomorphic to the ring of double-modular forms. Furthermore, I will compute the Lie algebra generated by the coordinate vector fields on T and show that it splits into a direct sum of two copies of genus zero congruence subgroups of $\mathrm{SL}(2,\mathbb{Z})$, depending on the type of elliptic fibre. This is work in progress with M. Alim.