Truth, its role and form in mathematics. A personal account.

Ralph M. Kaufmann

Dedicated to Yuri Ivanovich Manin on the occasion of his 85th Birthday.

The standard of truth in mathematical arguments is certainly subject to the times, and what constitutes a proof is both a function of time and a function of the speaker community. Euclid, Hilbert, Grothendieck, Bourbaki and Manin, are prime examples of one type of truth standard. This standard is not universally adhered to in practice. One can actually discern a movement away from this type of formalization. The advantage is that less formal and more intuitive handling of mathematical objects allows for greater creativity and faster communication. While this point can be conceded, what is at stake here is the Platonic character of mathematics and moreover certainty and independence. Famously for Plato, mathematics lies in between the real world and the world of ideas. This is what allows for a “higher truth” in mathematics. Counterintuitively, it is precisely this removedness of mathematics which makes it universal.

As much as one can appreciate the power of intuition and its creative role, the transparent style of data, axioms and clear-cut statements is unparalleled in its potential for understanding and conveying of mathematical truth. I personally encountered this while reading Quantum Groups and Noncommutative Geometry, which exemplifies just the right balance between motivation and precise abstract structures. The latter are what lends the aesthetic and the type of beauty that brings one closer to the world of ideas. In this form mathematical knowledge transcends and can become a metaphor, to borrow the phrase from Manin. Indeed, this is the reason Spinoza wrote his Ethics as “demonstrated in geometric order”.

The clarity of thought and the subsequent presentation created direct knowledge and provides structures that can become parts of thought patterns and provide structures of analysis or discourse. The laying out of the whole information to find the interconnections and to distill the essential structures from the accidental examples, while keeping the paradigmatic examples is a method that is exemplified by Yuri Ivanovich, and I am thankful to have experienced this firsthand under his guidance. This method has been a guide to me in my mathematical writing as well as in other subjects. Mathematically, Manin’s insight that “good proofs make us wiser” and the emphasis of programs versus results in mathematics weigh heavily. As he points out, a proof is a reduction to a tautology, but the content is not in the mere “Bedeutung”, but in the “Sinn”, to quote Frege.[1].

[1] Frege was incidentally, but maybe not coincidentally, the subject of a master thesis in philosophy I had written during my PhD studies in mathematics. The appreciation, yes Frege is worthwhile, and concern, nevertheless you should get back to mathematics, a stern “no more Frege”, that Yuri Ivanovich expressed when I had informed him about this after the fact, are maybe a “gelebte” manifestation of the presented positions, and for me epitomizes his character as an advisor.

One consequence of this is that the ontology of the problems becomes important. This is a possible answer to the question of what kind of proof is "sinnstiftend". To understand something, one must understand what the problem is on a higher level, which mathematically may be situated at a lower level. This is where Manin's work on logic is relevant. In this direction, the transferability to other areas is maybe best exemplified by Manin's early contemplation of quantum computers. A more mathematical example comes from associativity and commutativity equations and the coding of information, algebraic and geometric, in terms of categories and functors as one can find in Gelfand-Manin's "Homological Algebra", which I recommend to all my students. This view of geometry as sheaves combines Spinoza's "Geometric order" with Decartes' "Method"; the same principles underly geometry and algebra. This ties into the first, primordial, application of mathematics to the real world – physics. These three blocks are the fundament of Manin's eponymous seminar, and their conjunction is a hallmark of way of thinking which has been very influential and will certainly bear fruit well into the future.

There is perhaps a higher level of mathematics, a meta-mathematics, which just as meta-physics is outside the realm of its non-meta counterpart. This meta-mathematical level is important to be able to state what is "morally" true or why we know "things" are true before we prove them. Indeed, without knowing something is true and why it should be true one would not neither have a way to contemplate the "Sinn" nor to even start to tackle the problem or even to state it. This is the Promethean life-fire of mathematics, which must be separated from the subject matter itself. It belongs to the genesis and the personality of mathematics, but is distinct from the universality. There is a great democracy in mathematical writing in that it is impersonal and by being lifted from the individual it is available to everyone and hence can be timeless. Euclid's Elements is both astonishingly modern in its timelessness. This is what mathematical style and writing should ideally aspire to – like Greek statues, along with their Roman copies, still transfer beauty and truth and bring us closer to the world of ideas via their contemplation. Staying with this metaphor, of course the artwork would not exist without the artist and the timelessness of the pieces only attests to the mastery. In Yuri Ivanovich's case the mathematics and the character are perfectly aligned.[2]

I was very fortunate that Fortuna decided to afford me the possibility to study with Yuri Manin as my advisor. His guidance and example have allowed me to both see and appreciate the truth and beauty of mathematics and to contribute and pass along the methods and "Sichtweise" which Yuri Ivanovich aptly summarized as partaking in the common human endeavor to expand scientific knowledge and understanding, to which he unquestionably has and continues to fundamentally contribute.

---

[2] As a second footnote. The first time I met Yuri Ivanovich the image of Hesse's Siddhartha becoming wise while contemplating the river directly came to my mind. Seeing him and Xenia Glebova in their appartement on the Rhine is an almost prophetic fulfillment of this initial association.