ABSTRACTS

Bainbridge, Matthew (University of Chicago, US & MPI, Germany)

Algebraically primitive Teichműller curves in genus three

A Teichműller curve is a finite-volume Riemann surface C = H/G together with an isometric immersion C --> M_g, where M_g is the moduli space of genus g Riemann surfaces, equipped with the Teichműller metric. The trace field F(C) is the number field obtained by adjoining to Q the traces of the elements of G (a subgroup of SL_2(R)). It is known that F (C) has degree at most g, and C is said to be algebraically primitive if F(C) has degree g.

McMullen and Calta have independently discovered an infinite family of algebraically primitive Teichműller curvesin M_2. It is an open question whether there are infinitely many algebraically primitive Teichműller curves in M_g for any g>2. In this talk I will present some recent theorems and computer evidence indicating that there should be only finitely many algebraically primitive Teichműller curves in M_3. This is join work with Martin Möller.

Bufetov, Alexander (Rice University, US & MPI, Germany)

Limit theorems for translation flows

Consider a compact oriented surface of genus at least two endowed with a holomorphic one-form. The real and the imaginary parts of the one-form define two foliations on the surface, and each foliation defines an area-preserving translation flow. By a Theorem of H.Masur and W.Veech, for a generic surface these flows are ergodic. The talk will be devoted to the speed of convergence in the ergodic theorem for translation flows. The main result, which extends earlier work of A.Zorich and G.Forni, is a multiplicative asymptotic expansion for time averages of Lipschitz functions. The argument, close in spirit to that of G.Forni, proceeds by approximation of ergodic integrals by special holonomy-invariant Hoelder cocycles on trajectories of the flows. In particular, a simple explicit construction is given for G. Forni's invariant distributions of translation flows. Generically, the dimension of the space of holonomy-invariant Hoelder cocycles is equal to the genus of the surface, and the ergodic integral of a Lipschitz function can be approximated by such a cocycle up to terms growing slower than any power of the time. The renormalization effectuated by the Teichmueller geodesic flow on the space of holonomy-invariant Hoelder cocycles allows one also to obtain limit theorems for translation flows: it is proved that along certain sequences of times ergodic integrals, normalized to have variance one, converge in distribution to a non-degenerate compactly supported measure. The argument uses a symbolic representation of translation flows as suspension flows over Vershik's automorphisms, a construction similar to one proposed by S.Ito.

Deninger, Christopher (Universität Münster & MPI, Germany)

Noncommutative Mahler measures and Ljapunov exponents

For an ergodic measure preserving action on a probability space, consider the corresponding crossed product von Neumann algebra. We calculate the Fuglede-Kadison determinant for the "algebraic" operators in this von Neumann algebra in terms of the Ljapunov exponents of an associated measurable cocycle. The proof is based on the theory of the Brown measure, recent work of Dykema and Schultz and an analysis of the proof of the multiplicative ergodic theorem. As an application one obtains formulas for the Fuglede-Kadison determinant of noncommutative polynomials in the von Neumann algebra of the discrete Heisenberg group previously obtained by Lind and Schmidt via entropy considerations. These determinants can also be viewed as non-commutative generalizations of the Mahler measures prominent from number theory and hence as non-commutative heights.

Einsiedler, Manfred (Ohio State University, US & ETH Zürich, Switzerland & MPI, Germany)

Diophantine approximation on invariant fractals

We start by recalling the conjectures and the known theorems regarding the dynamics of diagonalizable subgroup actions on homogeneous spaces. In particular, we will discuss the question which probability measures are invariant under the full diagonal subgroup A on SL(n,Z)\SL(n,R) for n>2 and the partial classification of such measures due to Katok, Lindenstrauss, and myself. In the same joint work this theorem was applied to Littlewood's conjecture on multiplicative Diophantine approximation. Somewhat surprisingly, the same (or closely related) theorems also help to understand the Diophantine properties of typical points on dynamically defined fractals like the regular middle-third Cantor set. We will discuss why almost every point on the Cantor set is well approximable and related theorems. This is joint work with Fishman and Shapira.

Glasner, Eli (Tel-Aviv University, Israel & MPI, Germany)

A topological lens for a measure-preserving system

I will describe a functor which associates to every measure preserving system $(X,\mathcal{B},\mu,T)$ a topological system $(C^2(\mu),\check{T})$ defined on the space of 2-fold couplings of μ, called the *topological lens* of T. As we shall see the topological lens 'magnifies' the basic measure dynamical properties of T in terms of the corresponding topological properties of $\check{T}$. Some of our main results are as follows: (i) T is weakly mixing iff $\check{T}$ is topologically transitive (iff it is topologically weakly mixing); (ii) T has zero entropy iff $\check{T}$ has zero topological entropy, and T has positive entropy iff $\check{T}$ has infinite topological entropy; (iii) For T a K-system, the topological lens is a P-system (i.e. it is infinite, topologically transitive and the set of periodic points is dense; such sytems are also called chaotic in the sense of Devaney). This is a joint work with M. Lemanczyk and B. Weiss.

Gorodnik, Alexander (University of Bristol, UK & MPI, Germany)

Rigidity of actions on algebraic spaces

In this talk we will be interested in showing that objects that apriori are defined only in the measurable category have to respect geometric/ algebraic structure of the space. In particular, we discuss a classification of measurable morphisms and factors for dynamical systems on general algebraic varieties, which is analogous to the Margulis' classification of factors of flag varieties. This is a joint work with Bader, Furman, and Weiss.

Jones, Rafe (College of the Holy Cross, US)

Galois actions on preimage trees

In a discrete dynamical system, the set of all points eventually mapping to a given point P forms a rooted tree in a natural way. When the system is given by iteration of a rational function (or more generally a morphism) of degree d, and both the map and P are defined over a number field K, then there is a natural action of the absolute Galois group of K on the rooted tree of preimages of P. This action is known as an arboreal Galois representation. This talk presents a survey of known results about such representations, in particular addressing the question of when the action has finite index in the full automorphism group of the preimage tree. I will also give some applications to arithmetic questions about certain dynamical systems, and discuss analogies with the theory of linear Galois representations attached to abelian varieties.

Kontorovich, Alex (Brown University, US)

On representations of integers in thin subgroups of SL(2,Z)

The Affine Linear Sieve extends sieve methods to thin orbits of non-abelian group actions. The fundamental work of Bourgain-Gamburd-Sarnak showed in great generality that one may obtain an infinitude of R-almost primes (numbers with at most R prime factors), but the method does not specify R. Explicit values of R were obtained in some specific situations by Kontorovich and Kontorovich-Oh, but unconditionally are still in the teens (i.e. R=14). In recent joint work with Jean Bourgain, we obtain an infinitude of actual primes in such an orbit. We show that not only do infinitely many primes appear, but almost every number appears, as long as it is not excluded by congruence conditions; moreover the number of exceptions is a power savings. The main ingredient is the circle method.

Lindenstrauss, Elon (Hebrew University of Jerusalem, Israel & MPI, Germany)

Arithmetic combinatorics and effective equidistribution

A remarkable development in recent years has been a dramatic advance in our understanding of quantitative equidistribution thanks to the techniques of arithmetic combinatorics. I will present one result in this direction by Bourgain, Furman, Mozes and myself for random walks on the torus by endomorphisms, and explain how it relates to other results in the field, in particular an exponential sum estimate by Bourgain, Glibichuk and Konyagin.

Margulis, Gregori (Yale University, US)

Distribution of values of indefinite quadratic forms at integral points

I will describe different approaches to to the study of the aymptotic behavior of the number of integral solutions of the inequality a<Q(x)<b in "large" domains where Q is an indefinite irrational quadtaric form in n>2 variables.

Marklof, Jens (University of Bristol, UK & MPI, Germany)

The asymptotic distribution of Frobenius numbers and flows on homogeneous spaces

The Frobenius number F(a) of an integer vector a with positive coprime coefficients is defined as the largest number that does not have a representation as a positive integer linear combination of the coefficients of a. We show that if a is taken to be random in an expanding d-dimensional domain, then F(a) has a limit distribution, which is given by the probability distribution for the covering radius of a certain simplex with respect to a (d-1)-dimensional random lattice. This result extends recent studies for d=3 by Arnold, Bourgain-Sinai and Shur-Sinai-Ustinov. The key features of our approach are (a) a novel interpretation of the Frobenius number in terms of the dynamics of a certain group action on the space of d-dimensional lattices, and (b) an equidistribution theorem for a multidimensional Farey sequence on closed horospheres.

Nevo, Amos (Technion - Israel Institute of Technology & MPI, Germany)

Ergodic theory and lattice points

We will begin by describing a general lattice point counting result based on ergodic theoretic methods. We will then expose some of its consequences for counting integral, rational and (almost) prime points on certain homogeneous algebraic varieties.

Marmi, Stefano (Scuola Normale Superiore di Pisa, Italy & MPI, Germany)

Roth type interval exchange maps

In 1955 Roth proved that every irrational algebraic number is of Roth type, namely has irrationality measure equal to 2. Lesser known is that Roth type numbers have an equivalent characterization in terms of the regularity of the soultions of the cohomological equation associated to circle rotations. Interval exchange maps generalize rotations and are characterized by combinatorial and metric data. The analysis of first return times on an interval (renormalisation) leads to a remarkable extension of the classical continued fraction algorithm (Rauzy, Veech, Zorich). We exhibit an explicit full measure class of minimal interval exchange maps for which the cohomological equation has a bounded solution provided that the datum belongs to a finite codimension subspace of the space of functions with derivative of bounded variation on each interval. The class of interval exchange maps is characterized in terms of a diophantine condition of Roth type imposed to an acceleration of the Rauzy-Veech-Zorich continued fraction expansion associated to the map. This talk is based on joint work with Pierre Moussa and Jean-Christophe Yoccoz.

Oh, Hee (Brown University, US & MPI, Germany)

Orbital counting for Thin groups

For a given homogeneous variety V of a semisimple algebraic group G defined over Q, and for a subgroup Gamma of G(Z), the orbital counting problem for Gamma refers to the question of understanding the asymptotic number of integral vectors in the orbit v_0Gamma (for some fixed v_0 in V(Z)) of size at most T as T tends to infinity. This question has been extensively studied when Gamma is of finite index in G(Z). On the other hand, the orbital counting problem for a thin group (that is, when Gamma is of infinite index in G(Z)) is very far from being known in general. I will discuss a recent work with Nimish Shah on this question when V is a quadric defined by an integral quadratic form of signature (n,1), with n at least 2. If time permits, I will also explain an application to a hyperbolic circle packing problem.

Pollicott, Mark (University of Warwick, UK & MPI, Germany)

Random matrix products and absolute continuity of the associated measures

In this talk we will consider random products of a finite set of matrices in SL(2, R). For positive matrices we present a new algorithm for accurately evaluating their Lyapunov exponents and the Furstenberg (stationionary) measure. In joint work with B. Barany and K. Simon, we give some conditions for the measure to be absolutely continuous. Finally, we consider the statistics of these random products and, in joint work with R. Sharp, their extensions to group actions.

Schmidt, Klaus (University of Vienna, Austria & MPI, Germany)

Mahler measure, a dynamical number

This talk is about connections between orbit counts, entropy and Mahler measure for toral automorphisms and much more general algebraic actions of countable discrete groups. Expansiveness appears to play an important role in these connections, and many questions remain open in the absence of expansiveness.

Shapira, Uri (Hebrew University of Jerusalem, Israel & MPIM, Germany)

Applications of rigidity of higher rank hyperbolic actions

In this talk I will describe a general framework linking between dynamics on homogeneous spaces and Diophantine approximations and survey some recent results. In particular I will describe two families of problems: 1) Generalized Littlewood conjectures (GLC); 2) Inhomogeneous Minima problems (IM). Regarding problem (1) I will show that there are cases where GLC holds. Regarding problem (2) I will compute the IM in certain cases.

Applications of the results to Diophantine app' will be given and open problems will be presented.

Skau, Christian (Norwegian U. Scie. and Technology & MPI, Germany)

Orbit equivalence and (ordered) K-theory

To a dynamical system -- either in the measure-theoretic or in the topological setting -- one can associate an operator algebra. The invariants for operator algebras are of (ordered) K-theoretic nature,and this reflects back on the dynamical systems as invariants for orbit equivalence. We will give a survey of this aspect of studying minimal dynamical systems,and we will present some recent results.

Snoha, Lubomir (Matej Bel University, Slovakia & MPI, Germany)

Topology of minimal sets

Discrete dynamical systems given by a continuous map on a topological (usually compact metrizable) space will be considered. Minimality of such a system/map can be defined as the density of all forward orbits. Every compact system contains at least one minimal set, i.e., a nonempty closed invariant subset such that the restriction of the map to this subset is minimal. A fundamental question in topological dynamics is the one on the topological structure of minimal sets of continuous maps in a given space (and, in particular, whether the space itself admits a minimal map or not). In the first part of the talk we will present a survey of some known facts on minimality (including those on topological properties of minimal maps and on noninvertible minimal maps). Then we will concentrate on the topological structure of minimal sets. Among others, recent results obtained jointly with Kolyada and Trofimchuk and with Balibrea, Downarowicz, Hric and Spitalsky will be presented.

Solomyak, Boris (University of Washington, US & MPI, Germany)

Dynamics of self-affine tilings and algebraic number theory

We consider a self-affine tiling (with a diagonalizable expansion matrix) in the Euclidean space and the associated dynamical system, namely, the translation action on the orbit closure of the given tiling. We investigate the spectral properties of the system. It turns out that the presence of the discrete component depends on the algebraic properties of the eigenvalues of the expanding matrix for the tiling. In particular, we prove that the system is weakly mixing if and only if the matrix is totally non-Pisot. The proof is based on a kind of "rigidity" for self-affine tilings and uses some techniques of Thurston and Kenyon. This is a joint work with Jeong-Yup Lee.

Vershik, Anatoly (St. Petersburg Dep.Steklov In-te Mathematics, Russia)

Arithmetic and dynamics

We will discuss the link between the ergodic theory and reordering of the integers. In the general part we will discuss how to retell dynamics in terms of infinite permutations of the naturals. The concrete part of the talk will consist in the discussion on well-known Morse transformation from that point of view. We define a new partial order on the 2-adic numbers $\mathbf{Z}^2$ which can be extend to additive group of all didadic numbers $\mathbf{Q}^2$ and extend on the upper-triangle group of SL(2, $\mathbf{Q}^2$).

Ugarcovici, Ilie (DePaul University, US & MPI, Germany)

Structure of attractors for (a,b)-continued fractions, II

This talk reports on a joint work with Svetlana Katok and will continue the presentation from last Tuesday about a two parameter family of continued fractions and the associated one-dimensional maps suggested for consideration by Don Zagier. There exist sufficient conditions for the natural extension maps to have domains with relatively simple geometric structure (finitely rectangular). We prove that the set of parameters for which the geometric

structure is not finitely rectangular belongs to the boundary of the parameter set, and show how one can construct infinitely many such counter-examples. When the domain of the natural extension map is finitely rectangular, we compute explicitly the absolutely continuous invariant measure of the associated interval map. We also address some possible applications of these continued fractions algorithms to coding geodesics on the modular surface.

Ulcigrai, Corinna (University of Bristol, UK & MPI, Germany)
Absence of mixing in locally Hamiltonian flows on surfaces

We consider a class of area-preserving (locally Hamiltonian) flows on a surface of genus g. We are interested in their ergodic properties, especially mixing: it turns out that the presence/absence of mixing depends on the type of fixed points. We had proved that the presence of centers in a generic such flow is enough to create mixing. Recently we showed that if the flow has only simple saddles, it is generically not mixing, but weakly mixing. The results use the representation of these flows as suspensions over interval exchange transformations and Diophantine-type of conditions on the interval exchange transformation.

Wang, Zhiren (Princeton University, US & HCM, Germany)
A quantitative result on actions of commutative toral automorphisms

Let G be an abelian subgroup of SL(d,Z). When G acts totally irreducibly on T^d and is not virtually-cyclic, Berend proved that every orbit on T^d is either the whole torus or finite. With the additional assumption that G is maximal among the subgroups satisfying Berend's condition, we give an effective statement of this result. This is an analogue of the recent quantitative Furstenberg's theorem concerning the x2, x3 action on the circle by Bourgain-Lindenstrauss-Michel-Venkatesh. If time allows we will also mention progress in the general case without the maximal rank assumption. This is part of my thesis work under the supervision of Prof. Elon Lindenstrauss.

Yoccoz, Jean-Christophe (Collège de France & MPI, Germany)
Affine interval exchange maps

I will report on jointwork with S.Marmi and P.Moussa on affine interval exchange maps. We show that for almost every standard interval exchange map T_0 of genus >1, and all possible slopes except for a codimension 1 subspace, any affine interval exchange map T semi-conjugated to T_0 has wandering intervals: in fact, its non-wandering set has zero Lebesgue measure.

Zieve, Michael (Institute for Advanced Study, US)
Intersections of polynomial orbits, and a dynamical Mordell-Lang conjecture

Pick nonlinear f,g in **C**[x]$, and arbitrary x,y in **C**. I will explain the following result: if the orbits {x, f(x), f(f(x)), ...} and {y, g(y), g(g(y)), ... } have infinite intersection, then f and g have a common iterate. The main ingredients in the proof are Siegel's theorem on integral points on curves, a specialization argument due to Lang, a result of Bilu and Tichy on diophantine equations with infinitely many solutions, and a new description of the set of expressions of a given polynomial as the functional composition of other polynomials. I will also discuss a general question which includes both this result and the Mordell-Lang conjecture.