

TITLE: Small eigenvalues of the Laplacian on surfaces

ABSTRACT: Eigenvalues of the Laplacian on hyperbolic surfaces are called small, if they lie below $1/4$, the bottom of the spectrum of the Laplacian on the hyperbolic plane. Buser showed that, for any $n \in \mathbb{N}$ and $\varepsilon > 0$, the closed surface S of genus $g \geq 2$ carries a hyperbolic metric with $2g - 2$ eigenvalues below ε and n eigenvalues below $1/4 + \varepsilon$. Buser's results were refined by Schmutz, and they conjectured that a hyperbolic metric on S has at most $2g - 2$ small eigenvalues. I will discuss this conjecture and related results, in particular the solution of the conjecture by Otal and Rosas and joint work of Matthiesen, Mondal, and myself in which we extend the results of Otal and Rosas.