KURDYKA-ŁOJASIEWICZ-SIMON INEQUALITY FOR GRADIENT FLOWS IN METRIC SPACES

DANIEL HAUER

Abstract. In this talk, I will present new tools and methods in the study of the trend to equilibrium of gradient flows in metric spaces $(\mathfrak{M}, d)$ in the *entropy* and *metric* sense, to establish *decay rates*, *finite time of extinction*, and to characterise *Lyapunov stable equilibrium points*. More precisely,

- I begin by introducing a gradient inequality in the metric space framework, which in the Euclidean space $\mathbb{R}^N$ is due to Łojasiewicz [Éditions du C.N.R.S., 87-89, Paris, 1963] and Kurdyka [Ann. Inst. Fourier, 48 (3), 769-783, 1998]. In the Hilbert space framework, this inequality is known under the name *Kurdyka-Łojasiewicz gradient inequality* and I will outline its connection to the classical *entropy-entropy production inequality* used in kinetic theory to study the long time asymptotic behaviour of flows (e.g., time-dependent probability distribution on the phase space of particles).
- I show that the validity of the Kurdyka-Łojasiewicz gradient inequality in a neighbourhood of an equilibrium point yields the trend to equilibrium in the *entropy sense* and *metric sense* and provides *decay rates* and *finite time of extinction* of gradient flows.
- I explain the construction of a *talweg curve* and show that it yields the validity of a Kurdyka-Łojasiewicz inequality with *optimal* growth function θ.
- For $1 < p < \infty$, I outline that on the p-Wasserstein space $\mathcal{P}_p(\mathbb{R}^N)$ or on $\mathcal{P}_{p,d}(X)$ provided (X, d, ν) is a (compact) measure length spaces satisfying a (p,∞)-*Ricci curvature bounded from below by* $K \in \mathbb{R}$, the Kurdyka-Łojasiewicz inequality becomes the celebrated *Talagrand entropy transportation inequality*. We show that this one is equivalent to p-logarithmic Sobolev inequalities. On $\mathcal{P}_{p,d}(X)$, this problem is connected with an interesting regularity problem. We note that our notion of Ricci curvature is consistent in the case $p = 2$ with the one introduced by Lott & Villani [Ann. Math. (2),169(3):903-991, 2009] and Sturm [Acta Math., 196(1):133-177, 2006].

The results presented in this talk are obtain in joint work with Prof José Mazón (Universitat de València, Valencia, Spain) and available at

http://www.maths.usyd.edu.au/u/pubs/publist/preprints/2017/hauer-7.html
or

https://arxiv.org/abs/1707.03129.

(Daniel Hauer) School of Mathematics and Statistics, The University of Sydney, NSW 2006, Australia
E-mail address: daniel.hauer@sydney.edu.au