A CHARACTERIZATION OF CODIMENSION 1 COLLAPSE UNDER BOUNDED CURVATURE AND DIAMETER

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Abstract: Let $\mathcal{M}(n, D)$ be the space of closed n-dimensional Riemannian manifolds (M, g) with $\operatorname{diam}(M) \leq D$ and $|\sec^M| \leq 1$. In this paper we consider sequences (M_i, g_i) in $\mathcal{M}(n, D)$ converging in the Gromov-Hausdorff topology to a compact metric space Y. We show on the one hand that the limit space of this sequence has at most codimension 1 if there is a positive number r such that the quotient $\frac{\operatorname{vol}(B_r^{M_i}(x))}{\operatorname{inj}^{M_i}(x)}$ can be uniformly bounded from below by a positive constant $C(n, r, Y)$ for all points $x \in M_i$. On the other hand, we show that if the limit space has at most codimension 1 then for all positive r there is a positive constant $C(n, r, Y)$ bounding the quotient $\frac{\operatorname{vol}(B_r^{M_i}(x))}{\operatorname{inj}^{M_i}(x)}$ uniformly from below for all $x \in M_i$. The proof uses results about the structure of collapse in $\mathcal{M}(n, D)$ by Cheeger, Fukaya and Gromov.