Lectures on *Ground states of quantum lattice systems*

QFT and Topological Phases via Homotopy Theory and Operator Algebras, Bonn, 6/30 - 7/3/25

References

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Lecture I

Quantum lattice systems: observables, dynamics, ground states.

0. For the basics: see section 1-3 John von Neumann Lectures, I wrote with Bob Sims.

1. Lectures by Daniel Spiegel on "C*-Algebraic Foundations of Quantum Spin Systems", at the Summer School on C*-Algebraic Quantum Mechanics and Topological Phases of Matter, University of Colorado Boulder, July 29 to August 2, 2024. for lecture notes and video recordings see here: `https://sites.google.com/colorado.edu/caqm`

2. Pieter Naaijkens, Quantum spin systems on infinite lattices (Springer Lecture notes in physics). Also available on arXiv: arXiv:1311.2717.

Lecture II

GNS representation, ground state gap, examples. John von Neumann Lectures

Lecture III

Lieb-Robinson bounds and locality; almost local observables [10, 7]. Quasi-adiabatic evolution [6, 5]. Automorphic equivalence [4, 1, 9].

Lecture IV

Stability of the ground state gap [2, 3, 8, 11]; the bulk gap [12]. Ogata's H^2 invariants [13, 14]. Examples.

References

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[2] S. Bravyi, M. Hastings, and S. Michalakis, *Topological quantum order: stability under local perturbations*, J. Math. Phys. **51** (2010), 093512.

[3] S. Bravyi and M. B. Hastings, *A short proof of stability of topological order under local perturbations*, Commun. Math. Phys. **307** (2011), 609.

[4] X. Chen, Z.-C. Gu, and X.-G. Wen, *Local unitary transformation, long-range quantum entanglement, wave function renormalization, and topological order*, Phys. Rev. B **82** (2010), no. 15, 155138.

[5] M. B. Hastings, *Lieb-Schultz-Mattis in higher dimensions*, Phys. Rev. B **69** (2004), 104431.

[6] M. B. Hastings and X. G. Wen, *Quasi-adiabatic continuation of quantum states: The stability of topological ground-state degeneracy and emergent gauge invariance*, Phys. Rev. B **72** (2005), 045141.

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[8] S. Michalakis and J.P. Zwolak, *Stability of frustration-free Hamiltonians*, Commun. Math. Phys. **322** (2013), 277–302.

[9] B. Nachtergaele, *From Lieb-Robinson bounds to automorphic equivalence*, The Physics and Mathematics of Elliott Lieb, Volume 2 (R.L. Frank, A. Laptev, M. Lewin, and R. Seiringer, eds.), European Mathematical Society, 2022, pp. 79–92.

[10] B. Nachtergaele, R. Sims, and A. Young, *Quasi-locality bounds for quantum lattice systems. I. Lieb-Robinson bounds, quasi-local maps, and spectral flow automorphisms*, J. Math. Phys. **60** (2019), 061101.

[11] ———, *Quasi-locality bounds for quantum lattice systems and perturbations of gapped ground states II. Perturbations of frustration-free spin models with gapped ground states*, Ann. H. Poincaré **23** (2022), 393–511.

[12] ———, *Stability of the bulk gap for frustration-free topologically ordered quantum lattice systems*, Lett. Math. Phys. **114** (2024), no. 24, 1–55.

[13] Y. Ogata, *A classification of pure states on quantum spin chains satisfying the split property with on-site finite group symmetries*, Trans. Amer. Math. Soc. Series B **8** (2021), 39–65, ArXiv:1908.08621.

[14] ———, *Classification of gapped ground state phases in quantum spin systems*, International Congress of Mathematicians 2022 (Dmitry Beliaev and Stanislav Smirnov, eds.), vol. V, European Mathematical Society, 2023, pp. 4142–4161.